Reliability Measures of Hybrid Electric Vehicles (HEVs) and Plug in Hybrid Electric Vehicles (PHEVs)

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Abstract

This study deals with the analysis of availability and reliability of hybrid electric vehicles (system 1) and plug-in hybrid electric vehicles (system 2). The purpose of this study is to find out the opinion of consumers who can afford their own hybrid car. The distribution of failure and repair rates is assumed to be exponential. A method of linear differential equations (LDE) is used to estimate reliability metrics such as average system failure and steady-state availability. Some special cases were evaluated using different values of the failure rates. In addition, we examined how the failure rate affected the system performances measures and we demonstrated the basic involved concept comparing the results of both systems. The results are also presented graphically using MATLAB software.

Keywords: *Reliability, Steady -State Availability, Linear Differential Equation, Mean Time to System Failure(MTSF).*

1. Introduction

In the subject of system reliability, a number of authors analyzed and assessed reliability matrices, including MTSF, steady-state availability, busy period of repairmen, and cost analysis with the Markov renewal process using regeneration point approach. In this work, dependability measures are assessed using linear differential equations technique. This approach is less complicated than the others, and MATLAB software can be used to carry out computations.Many authors previously employed linear differential equations techniques to evaluate reliability measures for various systems. The availability analysis and reliability measure of two non-identical systems were examined by El-Said et al. using the linear differential equation [1].El.sherbeny at analysed the behaviour of some industrial systems in light of the cost-free warranty policy [2].Gupta and Mittal investigated the stochastic behaviour of a two-unit warm standby system with two types of repairmen and varying levels of patience time [3]. Mokaddies et al. assessed the reliability and availability of two dissimilar-unit cold standby systems with three modes using a linear differential equation with no cost-benefit analysis [4]. Gao at el Studied a *K*-out-of M+W+C: G mixed standby system with an unreliable repair facility [5]. Uba Ahmad Ali et al. evaluated the dependability of a two different unit cold standby system with three modes using the Kolmogorov Forward equation approach [6]. Yusuf, I.et. al. studied Stochastic Modeling and performance measures of redundant system operating in different conditions[9]. Pradeep K. Joshi et al discussed the dependability and availability of a two-unit standby redundancy system using the linear differential equation solution technique [7].Lane et. al. analysed data from a survey of drivers (n = 1080) administered in late 2013 to assess factors that influence potential car buyers to consider two different types of plug-in electric vehicles (PEVs) in the United States: plug-in hybrid electric vehicles (PHEVs) and battery electric vehicles (BEVs) [8].The goal of this study is to investigate the dependability matrices, such as MTSF and steady state availability analysis, of a hybrid four-wheeler (system 1) and plug-in hybrid four-wheeler (system 2) using linear differential equation technique. Plug-in hybrid four-wheeler (System 2) is superior to hybrid four-wheeler(System 1) based on the computation presented in the study regarding the influence of the battery charging option and switching.A graphical representation of measures of system effectiveness of both the system is also explored.

2. Model Description and Assumptions

In this work, two types of electrical vehicles were studied: hybrid electric vehicles (system 1) and plug-in hybrid electric vehicles (system 2).

2.1 **Hybrid Electric Vehicle:** Hybrid electric vehicles are powered by an internal combustion engine and an electric motor which uses energy stored in batteries.A hybrid electric vehicle cannot be plugged in to charge the battery.

2.2 **Plug in Hybrid Electric Vehicle**: Plug in hybrid electric vehicle use batteries to power an electric motor, as well as another fuel such as gasoline or diesel to power an internal combustion engine or other propulsion sources. PHEV can charge their batteries through charging equipment and regenerative braking.

Throughout the study of research paper, the following assumptions has been made:

- System 1 (HEV) can generate electricity through regenerative braking rather than by plugging into a charging station to recharge the vehicle's battery.
- The hybrid four-wheeler in system 1 continues to run even if the battery failed.
- System 2 (PHEV) will only use its internal combustion engine as a backup and will be primarily driven by an electric motor.
- The switch in system 2 (PHEV) is utilized to turn on the petrol supply.
- System 2 (PHEV) uses an automatic switch to start the engine immediately when the battery dies (fails), provided the switch is in working order at the time of need; otherwise, the engine won't start until the switch is repaired.
- System 1 has only two modes, namely failure and normal.
- System 2 has three operating modes: normal, partial, and failure.
- Repair is flawless (as good as new)
- Only one change may be made at a time in a single state.
- All failure rates and repair rates are constant.

- Failure and repair rates are followed by an exponential distribution.

3 Notation and Symbol

S_i : Transition state of the system , i =0,1 , 2 ,3 ,4

P_N –Petrol supply Normal

B_N –Battery fully charged

B_{NP} –Battery partially charged

P_{NP} –Petrol supply is partially

B_F-Battery failed.

P_F-Petrol supply failed.

α −failure rate of petrol supply

β - failure rate of battery

α' -failure rate of petrol when battery already failed

β'-failure rate of battery when petrol already failed.

δ-Repair rate of Petrol supply

λ-Repair rate of battery

δ_1-rate of charging battery

α_2-rate of completion of battery charging

α_1-rate of filling petrol

μ_1-rate of completion of filling petrol

θ-replacement rate of both petrol &battery

4 Transition probability of Hybrid Electric Vehicle (System 1)

Figure 1 shows the transition probability of different states of system 1.

Up State ;$S_0 \equiv (B_N, P_N)$, $S_1 \equiv (B_N, P_F)$, $S_2 \equiv (B_F, P_N)$

Down State ; $S_3 \equiv (B_F, P_F)$

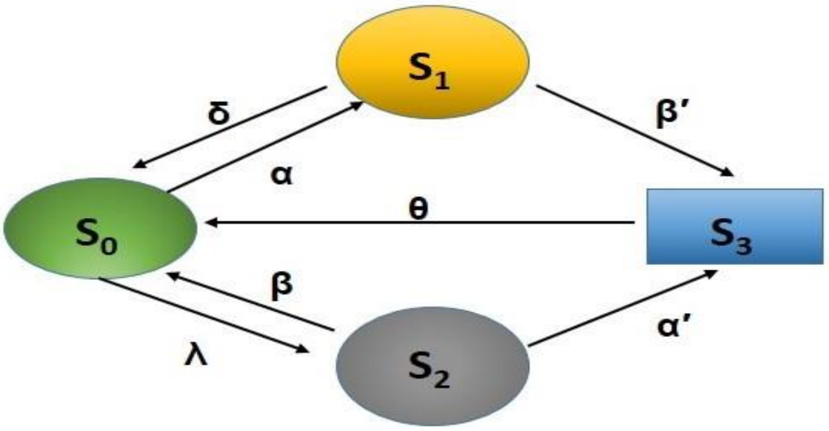


Figure 1. State Transition Diagram for the system1

5 Measures of System Effectiveness of System 1

5.1 Mean time to system failure (MTSF).

By applying linear differential equation technique and above assumptions, the mean time to system failure (MTSF) of the proposed system is determined. Define Pi(t) as the probability that the system is in state Si at time t, based on Figure 1. Let P(t) represent the probability row vector at time t. Consider the inditial conditions as :

$$P(0) = [\, P_0(0), P_1(0), P_2(0), P_3(0)], \; = [\, 1, 0 , 0 ,0 \,] \tag{1}$$

The derived system of differential equations is as follows :

$(dp_0(t))/dt$ = $-(\alpha+\beta)\, P_0(t) + \delta\, P_1(t) + \lambda\, P_2(t) + \theta P_3(t)$

$(dp_1(t))/dt$ = $-(\beta\text{^}' + \delta)P_1(t) + \alpha\, P_0(t)$

$(dp_2(t))/dt$ = $-(\alpha\text{^}' + \lambda\,)P_2(t) + \beta P_0(t)$

$(dp_3(t))/dt$ = $-\,\theta\, P_3(t) + \beta\text{^}'\, P_1(t) + \alpha\text{^}'\, P_2(t)$

which can be expressed in matrix form as

$$\frac{dp(t)}{dt} = A\,P \tag{2}$$

where

$$A = \begin{bmatrix} -(\alpha+\beta) & \delta & \lambda & \theta \\ \alpha & -(\beta'+\delta) & 0 & 0 \\ \beta & 0 & -(\alpha'+\lambda) & 0 \\ 0 & \beta' & \alpha' & -\theta \end{bmatrix}$$

We eliminate the rows and columns of the absorption state of the matrix A and transpose it to create a new matrix called Q because evaluating the transition solution is difficult. The expected time to reach an absorbing state is determined from

$E[T_{P(0) \to (absorbing)}] = P(0)\int_0^\infty e^{Qt}dt$

and

$\int_0^\infty e^{Qt}dt = -Q^{-1}$,since $Q^{-1} < 0$

where

$$Q = \begin{bmatrix} -(\alpha+\beta) & \alpha & \beta \\ \delta & -(\beta'+\delta) & 0 \\ \lambda & 0 & -(\alpha'+\lambda) \end{bmatrix}$$

The MTSF can be expressed explicitly as:

$$\text{MTSF}_1 = E[TP(0) \to (absorbing)] = P(0)(-Q^{-1})\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad (3)$$

$$\text{MTSF}_1 = \frac{(\alpha+\beta_1')(\lambda+\alpha')+\delta(\lambda+\alpha')+\beta(\delta+\beta')}{\alpha\beta'(\lambda+\alpha')+\beta\alpha'(\delta+\beta')} \qquad (4)$$

5.2 Steady -State Availability Analysis of the System 1

The initial condition for the availability analysis in Figure 1 is the same as for the reliability case.

$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0))] = [1, 0 , 0 ,0]$

The systemof differential equations can be expressed as:

$\dot{P} = A\,P$

$$\begin{bmatrix} \dot{p}_0 \\ \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \end{bmatrix} = \begin{bmatrix} -(\alpha+\beta) & \delta & \lambda & \theta \\ \alpha & -(\beta'+\delta) & 0 & 0 \\ \beta & 0 & -(\alpha'+\lambda) & 0 \\ 0 & \beta' & \alpha' & \theta \end{bmatrix} \begin{bmatrix} P_0 \\ p_1 \\ p_2 \\ P_3 \end{bmatrix}$$

The steady-state availability is given by

$A_{T_1}(\infty) = 1 - P_3(\infty)$ (5)

In the steady - state availability,the derivatives of the state probabilities become zero so that

$A\,P(\infty) = 0$ *(6)*

which is in matrix form

$$\begin{bmatrix} -(\alpha+\beta) & \delta & \lambda & \theta \\ \alpha & -(\beta'+\delta) & 0 & 0 \\ \beta & 0 & -(\alpha'+\alpha) & 0 \\ 0 & \beta' & \alpha' & \theta \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

$$P_0(\infty) + p_1(\infty) + P_2(\infty) + p_3(\infty) = 1 \tag{8}$$

To get $P_3(\infty)$ we substitute (8) in one of the redundant rows of (7) and use MATLAB to obtain the solution of the following system of linear equations in matrix form

$$\begin{bmatrix} -(\alpha+\beta) & \delta & \lambda & \theta \\ \alpha & -(\beta'+\delta) & 0 & 0 \\ \beta & 0 & -(\alpha+1) & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{9}$$

The solution of (9) provides the steady - state availability for Figure 1. The explicit expression for $AT_1(\infty)$is :

$$A_{T_1}(\infty) = \frac{\alpha(\alpha'+\lambda)(\beta'+\theta)+(\beta'+\delta)(\alpha'\beta+\theta\lambda)+\theta(\alpha'\beta'+\alpha\delta)}{\alpha(\alpha'+\lambda)(\beta'+\theta)+(\beta'+\delta)[\beta(\alpha'+\theta)+\theta(\alpha'+\lambda)]} \tag{10}$$

6. Transition probability of Plug- in hybrid electric vehicle(System 2)

Figure2 shows the transition probability of different states of system 2.

Up State ; $S_0 \equiv (B_N,P_N)$, $S_1 \equiv (B_{NP},P_N)$, $S_2 \equiv (B_N,P_{NP})$, $S_3 \equiv (B_N,P_F)$, $S_4 \equiv (B_F,P_N)$

Down State $S_5 \equiv (B_F, P_F)$

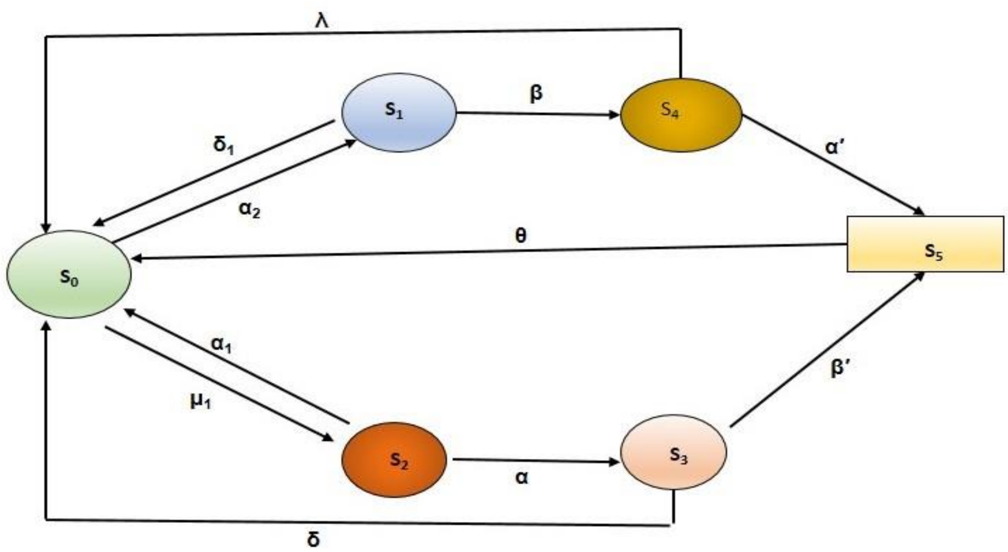


Figure 2; State transition diagram for the Second system

7 Measures of System Effectiveness of System 2

7.1 Mean Time to System Failure

By applying the linear differential equation technique and the above assumptions, the mean time to system failure (MTSF) of the proposed system is determined. Define Pi(t) as the probability that the system is in state Si at time t, based on Figure 2. Let P(t) represent the probability row vector at time t. Consider the inditial conditions as :

By applying the linear differential equation technique and the aforementioned assumptions, the mean time to system failure (MTSF) of the suggested system is determined. Define Pi(t) as the probability that the system will be in state Si at time t based on Figure 2. Let P(t) represent the probability row vector at time t. Consider the inditial conditions as :

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)] = [1,0,0,0,0,0] \tag{11}$$

The derived system of differential equations is as follows:

$$\frac{dp_0(t)}{dt} = -(\alpha_2 + \alpha_1)\, P_0(t) + \delta_1 P_1(t) + \mu_1\, P_2(t) + \delta P_3(t) + \lambda P_4(t) + \theta P_5(t)$$

$$\frac{dp_1(t)}{dt} = -(\beta + \delta_1)p_1(t) + \alpha_2 P_0(t)$$

$$\frac{dP_2(t)}{dt} = -(\alpha + \mu_1)P_2(t) + \alpha_1 P_0(t)$$

$$\frac{dP_3(t)}{dt} = -(\beta' + \delta)P_3(t) + \alpha P_2(t)$$

$$\frac{dP_4(t)}{dt} = -(\lambda + \alpha')P_4(t) + \beta P_1(t)$$

$$\frac{dP_5(t)}{dt} = -\theta P_5(t) + \beta' P_3(t) + \alpha' P_4(t)$$

which can be expressed in matrix form as

$$\dot{P} = A\, P \tag{12}$$

Where

$$A=\begin{bmatrix} -(\alpha_2+\alpha_1) & \delta_1 & \mu_1 & \delta & \lambda & 0 \\ \alpha_2 & -(\beta+\delta_1) & 0 & 0 & 0 & 0 \\ \alpha_1 & 0 & -(\alpha+\mu_1) & 0 & 0 & 0 \\ 0 & 0 & \alpha & -(\beta'+\delta) & 0 & 0 \\ 0 & \beta & 0 & 0 & -(\alpha'+\lambda) & 0 \\ 0 & 0 & 0 & \beta' & \alpha' & -\theta \end{bmatrix}$$

We eliminate the rows and columns of the absorption state of the matrix A and transpose it to create a new matrix called Q because evaluating the transition solution is difficult. The expected time to reach an absorbing state is determined from

$$E[T_{P(0)\rightarrow(absorbing)}] = P(0)\int_0^{\infty} e^{Qt}dt$$

and

$$\int_0^{\infty} e^{Qt}dt = -Q^{-1}, \text{since } Q^{-1} < 0$$

where

$$Q=\begin{bmatrix} -(\alpha_2+\alpha_1) & \alpha_2 & \alpha_1 & 0 & 0 \\ \delta_1 & -(\beta+\delta_1) & 0 & 0 & \beta \\ \mu_1 & 0 & -(\alpha+\mu_1) & \alpha & 0 \\ \delta & 0 & 0 & -(\beta'+\delta) & 0 \\ \lambda & 0 & 0 & 0 & -(\alpha'+\lambda) \end{bmatrix}$$

$$MTSF_2 = E[T_{P(0)\rightarrow(absorbing)}] = P(0)(-Q^{-1})\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} \quad (13)$$

$$MTSF_2 = \frac{\alpha_2\beta\alpha'(\alpha+\mu_1)(\delta+\alpha')+\alpha_1\alpha\beta'(\lambda+\alpha')(\beta+\delta_1)}{(\lambda+\alpha')[(\alpha+\mu_1+\alpha_1)(\beta+\delta_1)(\delta+\beta')+\alpha_1\alpha(\beta+s_1)]+\alpha_2(\delta+\beta')(\alpha+\mu_1)(\lambda+\alpha'+\beta)} \quad (14)$$

7.2 Steady **-State Availability Analysis of the System**

The initial condition for the availability analysis in Figure 2 is the same as for the reliability case.

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)] = [1,0,0,0,0,0]$$

The system of differential equation can be expressed as

$$\begin{bmatrix}\dot{p}_0\\ \dot{p}_1\\ \dot{p}_2\\ \dot{p}_3\\ \dot{p}_4\\ \dot{p}_5\end{bmatrix}=\begin{bmatrix}-(\alpha_2+\alpha_1) & \delta_1 & \mu_1 & \delta & \lambda & 0\\ \alpha_2 & -(\beta+\delta_1) & 0 & 0 & 0 & 0\\ \alpha_1 & 0 & -(\alpha+\mu_1) & 0 & 0 & 0\\ 0 & 0 & \alpha & -(\beta'+\delta) & 0 & 0\\ 0 & \beta & 0 & 0 & -(\alpha'+\lambda) & 0\\ 0 & 0 & 0 & \beta' & \alpha' & -\theta\end{bmatrix}\begin{bmatrix}p_0\\ p_1\\ p_2\\ p_3\\ p_4\\ p_5\end{bmatrix}$$

The steady-state availability is given by

$A_{T_2}(\infty) = 1 - P_5(\infty)$*(15)*

In the steady - state availability, the derivatives of the state probabilities become zero so that

$A\,P(\infty) = 0$

which in matrix form

$$\begin{bmatrix}-(\alpha_2+\alpha_1) & \delta_1 & \mu_1 & \delta & \lambda & 0\\ \alpha_2 & -(\beta+\delta_1) & 0 & 0 & 0 & 0\\ \alpha_1 & 0 & -(\alpha+\mu_1) & 0 & 0 & 0\\ 0 & 0 & \alpha & -(\beta'+s) & 0 & 0\\ 0 & \beta & 0 & 0 & -(\alpha'+\lambda) & 0\\ 0 & 0 & 0 & \beta^1 & \alpha^1 & -\theta\end{bmatrix}\begin{bmatrix}p_1(\infty)\\ p_1(\infty)\\ p_2(\infty)\\ p_3(\infty)\\ p_4(\infty)\\ p_5(\infty)\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix} \quad (16)$$

$$P_0(\infty)+p_1(\infty)+P_2(\infty)+p_3(\infty)+p_4(\infty)+p_5(\infty)=1 \quad (17)$$

To get$P_5(\infty)$ we substitute (17) in one of the redundant rows of (16) and use MATLAB to obtain the solution of the following system of linear equations in matrix form.

$$\begin{bmatrix}-(\alpha_2+\alpha_1) & s_1 & \mu_1 & \delta & \lambda & 0\\ \alpha_2 & -(\beta+\delta_1) & 0 & 0 & 0 & 0\\ \alpha_1 & 0 & -(\alpha_1) & 0 & 0 & 0\\ 0 & 0 & \alpha & -(\beta'+\beta) & 0 & 0\\ 0 & \beta & 0 & 0 & -(\alpha'+\lambda) & 0\\ 1 & 1 & 1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix}p_0(\infty)\\ p_1(\infty)\\ p_2(\infty)\\ p_3(\infty)\\ p_4(\infty)\\ p_5(\infty)\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix} \quad (18)$$

The solution of (18) provides the steady state probabilities in the availability case for Figure 2.The explicit expression for $AT_2(\infty)$ is :

$$AT_2(\infty) = \frac{\theta(\lambda+\alpha')(\alpha+\mu_1)(\beta+\delta_1)(\delta+\beta')+\alpha_2\theta(\alpha+\mu_1)(\delta+\beta')(\lambda+\alpha'+\beta) +\alpha_1\theta(\lambda+\alpha')(\beta+\delta_1)(\delta+\beta'+\alpha)}{2\beta(\delta+\beta')(\theta+\alpha')(\alpha+\mu_1)+\alpha_1\beta'(\lambda+\alpha')(\beta+\delta_1)(\alpha+\theta)+ \delta\theta(\alpha_2+\beta)(\lambda+\alpha')(\alpha+\mu_1)+\delta_1\theta(\delta+\beta')(\lambda+\alpha')(\alpha+\mu_1)+ \alpha_1\theta(\beta+\delta_1)(\lambda+\alpha')(\alpha+\delta)+\beta'\theta(\alpha_2+\beta)(\alpha+\mu_1)(\lambda+\alpha')} \quad (19)$$

8. Result and Discussion

We plot the MTSF and Steady State Availability for each model versus α and β, respectively, while keeping the other parameters fixed at $\alpha' = 1.5, \beta' = 1.5$, $\delta = 2.5$, $\lambda = 2$, $\mu_1 = \alpha_2 = 2.5$, $\delta_1 = \alpha_1 = 3$, $\theta = 3.5$ in order to observe the behavior of the system. We take $\beta = 0.4$ for the curve against α, and we take $\alpha = 0.4$ in addition to other parameters for the curve against β.

Table 1 The relationship between first and second system's availability, MTSF, and failure rate "α"

α	**MTSF₁**	**MTSF₂**	$AT_1(\infty)$	$AT_2(\infty)$
0.1	5.44	17.72	0.904	0.984
0.2	4.72	14.26	0.907	0.980
0.3	4.18	12.05	0.910	0.976
0.4	3.78	10.51	0.912	0.973
0.5	3.45	9.39	0.915	0.970
0.6	3.18	8.53	0.917	0.967
0.7	2.97	7.84	0.918	0.965
0.8	2.79	7.30	0.921	0.962
0.9	2.63	6.84	0.923	0.960
1.0	2.50	6.46	0.924	0.957

Table 2: The relationship between the first and second system's availability, MTSF, and failure rate "β"

β	**$MTSF_1$**	**$MTSF_2$**	$AT_1(\infty)$	$AT_2(\infty)$
0.1	5.85	46.8	0.975	0.982
0.2	4.90	29.7	0.953	0.979
0.3	4.25	22.11	0.932	0.976
0.4	3.78	17.81	0.913	0.973
0.5	3.41	15.05	0.894	0.971
0.6	3.12	13.12	0.870	0.968
0.7	2.89	11.83	0.860	0.966
0.8	2.70	10.61	0.840	0.964
0.9	2.53	9.75	0.830	0.962
1.0	2.40	9.01	0.820	0.960

Figure 3;MTSF against α

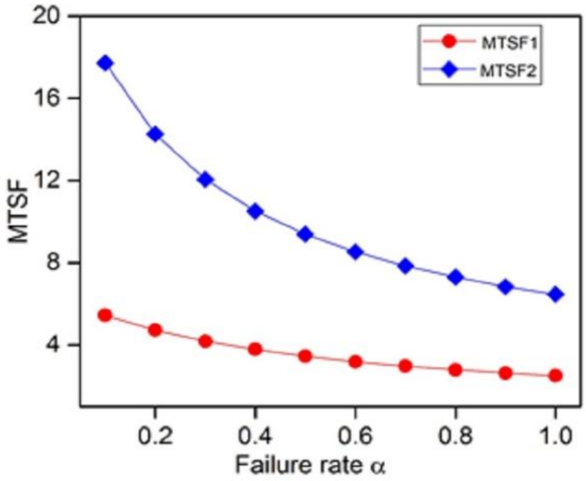


Figure 4; Availability against α

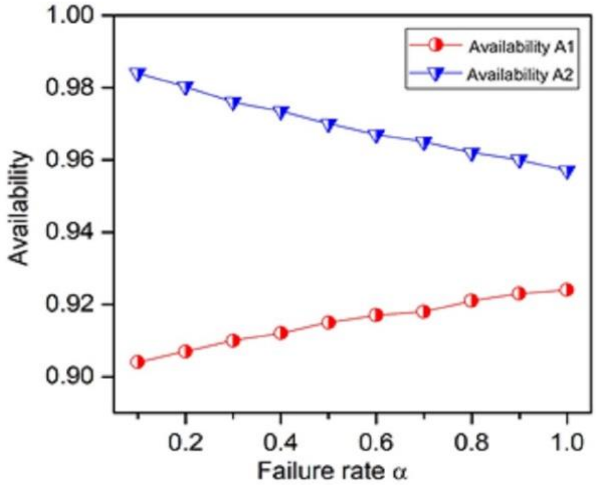


Figure 5; MTSF against β

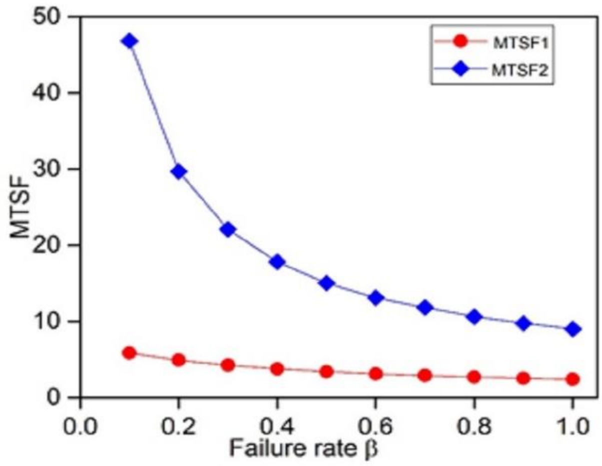
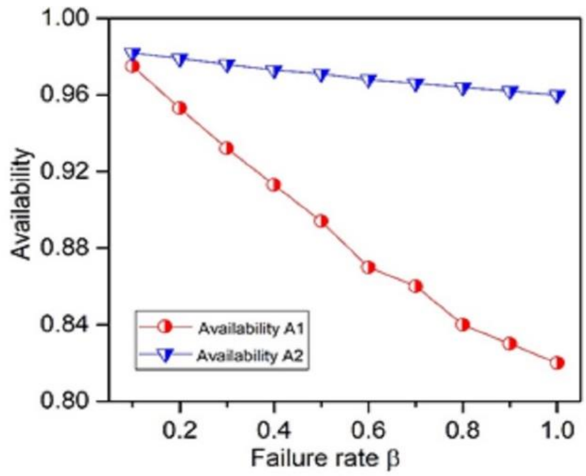


Figure 6; availability against β



As the value of α increases, the MTSF and availability of both systems decline, whereas the availability of system 1 marginally increases, as seen in figures 3 and 4. The graph makes it evident that system 2 has a higher mean time to system failure and availability than system 1.

The mean time to system failure and availability results for the two systems under study are plotted against the failure rate β in Figures 5 and 6.The figure clearly shows that, in comparison to system 1, system 2 has a higher mean time to system failure and availability.

Conclusion

This work uses Linear Differential Equation Techniques to analyze the availability and reliability of suggested systems, namely Plug-in Hybrid Electrical Vehicles (System 2) and Hybrid Electrical Vehicles (System 2). The impact of failure rate on both system's MTSF and steady state availability is also monitored in order to observe system behavior. From Figures 3 to 6, we can conclude that plug-in hybrid electrical vehicles (System 2) with switching and battery charging options have higher MTSF and availability than Hybrid Electric Vehicles (System 1), which do not have battery charging options. Therefore, Plug-in Hybrid Electrical Vehicles are preferable to hybrid electric vehicles.

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