A METHOD FOR SOLVING FUZZY TRANSPORTATION PROBLEM

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ABSTRACT: The most important and successful applications in the optimization refers to transportation problem. The main aspect of this paper is to find the least transportation cost of some commodities through a capacitated network when supply and demand of nodes and capacity and cost of edges are representedas fuzzy numbers. Here, we are solving the transportation problem using the Robust ranking technique, where fuzzy demand and supply are in the form of trapezoidal fuzzy numbers. The fuzzification of the cost of the transportation problem is discussed with the help of a numerical example.

KEY WORDS: Trapezoidal fuzzy numbers, Fuzzy transportation problem, Robust ranking technique.

1. Introduction

The transportation problem is one of the earliest applications of linear programming problems. Transportation models have wide applications in logistics and supply chain for reducing the cost effective algorithms have been developed for solving the transportation problem when the cost coefficients and supply and demand quantities are known exactly. The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. There are cases that the cost coefficients and supply and demand quantities of a transportation problem may be uncertain due to some uncontrollable factors. To deal quantitatively with imprecise information in making decisions Bellman et.al.,[2] introduced the notion of fuzziness.

Let a_i be the number of units of a product available at origin i and b_j be the number of units of the product required at destination j. Let C_{ij} be the cost of transporting one unit from origin i to destination j and let X_{ij} be the amount of quantity carried or shipped from origin i to destination j. There are effective algorithms for solving the transportation problems when all the decision parameters, i.e. the supply available at each source, demand required at each destination and unit transportation costs are given in a precise way. But in real life, there are many diverse situations due to uncertainty in one or more decision parameters and hence they may not be expressed in a precise way. This is due to measurement inaccuracy, lack of evidence, computational errors, high information cost, weather conditions etc. Hence we cannot apply the traditional classical methods to solve the transportation problems successfully. Therefore the use of Fuzzy transportation problems is more appropriate to model and solve the real world problems. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand are fuzzy quantities.

Bellman and Zadeh [3] proposed the concept of decision making in Fuzzy environment. After this pioneering work, several authors such as Shiang-Tai Liu and Chiang Kao[16], Chanas et al[5], Pandian et.al [14], Liu and Kao [11] etc proposed different methods for the solution of Fuzzy transportation problems. Chanas and Kuchta [4] proposed the concept of the optimal solution for the Transportation with Fuzzy coefficient expressed as Fuzzy numbers. Chanas, Kolodziejckzy, Machaj[5] presented a Fuzzy linear programming model for solving Transportation problem. Liu and Kao [11] described a method to solve a Fuzzy Transportation problem based on extension principle. Lin introduced a genetic algorithm to solve Transportation with Fuzzy objective functions. Srinivasan [18] - [23] described the new methods to solve fuzzy transportation problem.

Nagoor Gani and Abdul Razak [13] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. A.Nagoor Gani, Edward Samuel and Anuradha [7] used Arshamkhan"s Algorithm to solve a Fuzzy Transportation problem. Pandian and Natarajan [14] proposed a Fuzzy zero point method for finding a Fuzzy optimal solution for Fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers.

2. PRELIMINARIES

In this section we define some basic definitions which will be used in this paper.

2.1 Definition – 1

The characteristic function $\mu_A(x)$ of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}(x)$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_A: X \to [0,1]$. The assigned value indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}(x)$ is called the membership function and the set $\tilde{A} = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_{\tilde{A}}(x) \in [0,1]\}$. is called a fuzzy set.

2.2 Definition – 2

A fuzzy set A, defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_A : R \to [0,1]$ has the following characteristics

(i) A is normal. It means that there exists an x $\in$ R such that $\mu_A(x) = 1$

(ii) A is convex. It means that for every $x_1, x_2 \in R$,

$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\},\ \lambda \in [0,1]$

(iii) μ_A is upper semi-continuous.

(iv) supp (A) is bounded in R.

2.3 Definition – 3

A fuzzy number A is said to be non-negative fuzzy number if and only if $\mu_A(x) = 0,\ \forall\ x < 0$

2.4 Definition – 4

A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by, where a≤ b≤ c≤d.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a < x \leq b, \\ 1, & b < x < c, \\ \frac{d-x}{d-c}, & c \leq x < d, \\ 0, & x > d \end{cases}$$

2.5 Definition – 5

A trapezoidal fuzzy number $\tilde{A} = (a,b,c,d)$ is said to be non-negative (non - positive) trapezoidal fuzzy number. i.e. $A \geq 0 (A \leq 0)$ if and only if $a \geq 0 (c \leq 0)$. A trapezoidal fuzzy number is said to be positive (negative) trapezoidal fuzzy number i.e. $A > 0 (A < 0)$ if and only if $a > 0 (c < 0)$.

2.6 Definition – 6

Two trapezoidal fuzzy number $\tilde{A}_1 = (a,b,c,d)$ and $\tilde{A}_2 = (e,f,g,h)$ are said to be equal. i.e. $\tilde{A}_1 = \tilde{A}_2$ if and only if a=e,b=f,c=g,d=h.

2.7 Definition – 7

Let $\tilde{A}_1 = (a,b,c,d)$ and $\tilde{A}_2 = (e,f,g,h)$ be two non-negative trapezoidal fuzzy number then

(i) $\tilde{A}_1 \oplus \tilde{A}_2$ = (a,b,c,d) ⊕ (e,f,g,h) = (a+e,b+f,c+g,d+h)
(ii) $\tilde{A}_1 - \tilde{A}_2$ = (a,b,c,d)− (e,f,g,h) = (a-h,b-g,c-f,d-e)
(iii) $-\tilde{A}_1$ = - (a,b,c,d) = (-d,-c,-b,-a)
(iv) $\tilde{A}_1 \otimes \tilde{A}_2$ = (a,b,c,d) ⊗ (e,f,g,h) = (ae,bf,cg,dh)
(v) $\frac{1}{A} \cong (\frac{1}{d}, \frac{1}{c}, \frac{1}{b}, \frac{1}{a})$

2.8 Rou bast Ranking Technique

Rou bast ranking technique which satisfy compensation, linearity, and additivity properties and provides results which are consist of human intuition. If ã is a fuzzy number then the

Rou bast ranking is defined by $R(\tilde{a}) = \int_0^1 0.5(a_\alpha^L\ a_\alpha^U)d\alpha$, Where $(a_\alpha^L\ a_\alpha^U)$ is the α level cut of the fuzzy number $\tilde{a}$ and $(a_\alpha^L\ a_\alpha^U)$ = {((b-a)α+a),(d-(d-c)α)}

In this paper we use this method for ranking the objective values. The Rou bast ranking index $R(\tilde{a})$ gives the representative value of fuzzy number $\tilde{a}$.

3. Mathematical formulation of a fuzzy transportation problem

Mathematically a transportation problem can be stated as follows:

Minimize

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \quad \text{------(1)}$$

Subject to

$$\left.\begin{array}{ll} \sum_{j=1}^{n} x_{ij} = a_i & j = 1,2,\ldots,n \\ \sum_{i=1}^{m} x_{ij} = b_j & i = 1,2,\ldots,m \\ x_{ij} \geq 0 & i = 1,2,\ldots,m,\ j = 1,2,\ldots,n \end{array}\right\} \text{----(2)}$$

Where c_{ij} is the cost of transportation of an unit from the i^{th} source to the j^{th} destination, and the quantity x_{ij} is to be some positive integer or zero, which is to be transported from the i^{th} origin to j^{th} destination. A obvious necessary and sufficient condition for the linear programming problem given in (1) to have a solution is that

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j \quad \text{……………………… (3)}$$

(i.e) we assume that total availability is equal to the reequipment. If it is not true, a fictitioussource or destination can be added. It should be noted that the problem has feasible solution if
and only if the condition (2) satisfied. Now, the problem is to determine x_{ij}, in such a way that the total transportation cost is minimum

Mathematically a fuzzy transportation problem can be stated as follows:

Minimize

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \quad \text{…………………………………(4)}$$

Subject to

$$\left.\begin{array}{lll} \sum_{j=1}^{n} x_{ij} = \tilde{a}_i & j = 1, 2, \dots, n & \\ \sum_{i=1}^{m} x_{ij} = \tilde{b}_j & i = 1, 2, \dots, m & \\ x_{ij} \geq 0 & i = 1, 2, \dots, m, & j = 1, 2, \dots, n \end{array}\right\} \quad \text{---(5)}$$

In which the transportation costs $\tilde{c}_{ij}$, supply a_i and demand $\tilde{b}_j$ quantities are fuzzy quantities. An obvious necessary and sufficient condition for the fuzzy linear programming problem give in (4-5) to have a solution is that

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} \tilde{b}_j \quad \dots \quad \text{.......(6)}$$

This problem can also be represented as follows:

	1	………	n	Supply
1	$\tilde{c}_{11}$	………	$\tilde{c}_{1n}$	a_1
. . . .	. . . .	. . . .	. . . .	. . . .
M	$\tilde{c}_{m1}$	……....	$\tilde{c}_{mn}$	a_m
Demand	$\tilde{b}_1$	………	$\tilde{b}_n$	

4. Alternate Method for Solving Transportation Problem

Following are the steps for solving Transportation Problem

Step – 1: From the given Transportation problem, convert fuzzy values to crisp values using ranking function.

Step – 2: Deduct the minimum cell cost from each of the cell cost of every row/column of the Transportation problem and place them on the right-top/right-bottom of corresponding cost.

Step – 3: Adding the cost of right-top and right – bottom and place the summation value in the corresponding cell cost.

Step – 4: Identify the minimum element in each row and column of the Transportation table and subtract on their corresponding the row and the column.

Step – 5: Find the sum of the values in the row and the column. Choose the maximum value and allocate the minimum of supply/demand in the minimum element of the row and column. Eliminate by deleting the columns or rows corresponding to where the supply or demand is satisfied.

Step – 6: Continue step -4 and step -5 until of all the supply and demand is met.Step – 7: Place the original transportation cost to satisfied cell cost.

Step – 8: Calculate the minimum cost.

That is,

$$\text{Total Cost} = \sum\sum C_{ij} X_{ij}$$

5. Numerical Example

Consider the Fuzzy Transportation Problem

	FD_1	**FD_2**	**FD_3**	**FD_4**	**Fuzzy Capacity**
FO_1	[1,2,3,4]	[1,3,4,6]	[9,11,12,14]	[5,7,8,11]	[1,6,7,12]
FO_2	[0,1,2,4]	[-1,0,1,2]	[5,6,7,8]	[0,1,2,3]	[0,1,2,3]
FO_3	[3,5,6,8]	[5,8,9,12]	[12,15,16,19]	[7,9,10,12]	[5,10,12,17]
Fuzzy Demand	[5,7,8,10]	[1,5,6,10]	[1,3,4,6]	[1,2,3,4]	

Solution

In Conformation to model the fuzzy transportation problem can be formulated in the following mathematical programming form

Min Z = R(1,2,3,4)x_{11} + R(1,3,4,6)x_{12} + R(9,11,12,14)x_{13} + R(5,7,8,11)x_{14} +R(0,1,2,4)x_{21}+R(-1,0,1,2)x_{22}+R(5,6,78)x_{23}+R(4,5,6,7)x_{24}+R(3,5,6,8)x_{31}+R(5,8,9,12)x_{32} +R(12,15,16,19)x_{33} + R(7,9,10,12)x_{34}

$$R(\tilde{a}) = \int_0^1 0.5(a^L_\alpha \; a^U_\alpha) d\alpha$$

where (a^L_α, a^U_α) = {((b-a)+a),(d-(d-c))}

After applying ranking technique, we get

Table -1

	D1	**D2**	**D3**	**D4**	**Supply**
O1	2.5	3.5	11.5	7.75	6.5
O2	1.75	0.5	6.5	1.5	1.5
O3	5.5	8.5	15.5	9.5	11
Demand	7.5	5.5	3.5	2.5	

Table – 2

	D1	**D2**	**D3**	**D4**	**Supply**
O1	2.5^{0}_{0}	3.5^{1}_{3}	11.5^{9}_{5}	$7.75^{5.25}_{6.25}$	6.5
O2	$1.75^{1.25}_{0.75}$	$.5^{0}_{0}$	6.5^{6}_{0}	1.5^{1}_{0}	1.5
O3	5.5^{0}_{3}	8.5^{3}_{8}	15.5^{10}_{9}	9.5^{4}_{8}	11
Demand	7.5	5.5	3.5	2.5	

Table – 3

	D1	**D2**	**D3**	**D4**	**Supply**
O1	0	4	14	11.5	6.5
O2	2	0	6	1	1.5
O3	3	11	19	12	11
Demand	7.5	5.5	3.5	2.5	

With the help of the method, we get

Table – 4

	D1	**D2**	**D3**	**D4**	**Supply**
O1	0	5.5 4	1 14	11.5	6.5
O2	2	0	1.5 6	1	1.5
O3	7.5 3	11	1 19	2.5 12	11
Demand	7.5	5.5	3.5	2.5	

Finally, we get

Table – 5

	D1	**D2**	**D3**	**D4**	**Supply**
O1		**5.5** **3.5**	**1** **11.5**		6.5
O2			**1.5** **6.5**		1.5
O3	**7.5** **5.5**		**1** **15.5**	**2.5** **9.5**	11
Demand	7.5	5.5	3.5	2.5	

Hence (4+3-1)=6 cells are allocated and hence we got our feasible so ln next we calculate total cost and its corresponding allocated value of supply and demand

Total Cost (5.5X7.5) + (3.5X5.5) + (11.5X1) + (6.5X1.5) + (15.5X1) + (9.5X2.5) = 121

This is a basic feasible solution. The solution obtained using NCM, LCM, VAM and MODI/Stepping stone methods respectively. Hence the basic feasible solution obtained from method is optional soln.

6. Conclusion

In this paper, the transportation costs are considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy transportation problem of trapezoidal fuzzy numbers has been transformed into crisp transportation problem using robust ranking technique indices. Numerical examples show that by this method we can have the optimal solution as well as the crisp and fuzzy optimal total cost. By using Rou bast ranking method we have shown that the total cost obtained is optimal. Hence, this will be helpful for decision makers who are handling logistic and supply chain problems in fuzzy environment. For future research we propose effective implementation of the trapezoidal fuzzy numbers in all fuzzy problems.

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