Volume 1 Issue 1 Online ISSN–Applied

February 2025

RNT JOURNAL OF NEW TRENDS IN SCIENCE

Published By: DEEKSHA CREATIONS SOCIETY
4, Jai Shree Colony, Near Dhulkot
Udaipur-313001 (Rajasthan)

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Volume 1 Issue 1 January 2025

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Research Article

A METHOD FOR SOLVING FUZZY TRANSPORTATION PROBLEM

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ABSTRACT: The most important and successful applications in the optimization refers to transportation problem. The main aspect of this paper is to find the least transportation cost of some commodities through a capacitated network when supply and demand of nodes and capacity and cost of edges are represented as fuzzy numbers. Here, we are solving the transportation problem using the Robust ranking technique, where fuzzy demand and supply are in the form of trapezoidal fuzzy numbers. The fuzzification of the cost of the transportation problem is discussed with the help of a numerical example.

KEY WORDS: Trapezoidal fuzzy numbers, Fuzzy transportation problem, Robust ranking technique.

1. Introduction

The transportation problem is one of the earliest applications of linear programming problems. Transportation models have wide applications in logistics and supply chain for reducing the cost effective algorithms have been developed for solving the transportation problem when the cost coefficients and supply and demand quantities are known exactly. The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. There are cases that the cost coefficients and supply and demand quantities of a transportation problem may be uncertain due to some uncontrollable factors. To deal quantitatively with imprecise information in making decisions Bellman et.al.,[2] introduced the notion of fuzziness. Let a_i be the number of units of a product available at origin i and b_j be the number of units of the product required at destination j . Let C_{ij} be the cost of transporting one unit from origin i to destination j and let X_{ij} be the amount of quantity carried or shipped from origin i to destination j There are effective algorithms for solving the transportation problems when all the decision parameters, i.e. the supply available at each source, demand required at each destination and unit transportation costs are given in a precise way. But in real life, there are many diverse situations

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due to uncertainty in one or more decision parameters and hence they may not be expressed in a precise way. This is due to measurement inaccuracy, lack of evidence, computational errors, high information cost, weather conditions etc. Hence we cannot apply the traditional classical methods to solve the transportation problems successfully. Therefore the use of Fuzzy transportation problems is more appropriate to model and solve the real world problems. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand are fuzzy quantities.

Bellman and Zadeh [3] proposed the concept of decision making in Fuzzy environment. After this pioneering work, several authors such as Shiang-Tai Liu and Chiang Kao[16], Chanas et al[5], Pandian et.al [14], Liu and Kao [11] etc proposed different methods for the solution of Fuzzy transportation problems. Chanas and Kuchta [4] proposed the concept of the optimal solution for the Transportation with Fuzzy coefficient expressed as Fuzzy numbers. Chanas, Kolodziejckzy, Machaj[5] presented a Fuzzy linear programming model for solving Transportation problem. Liu and Kao [11] described a method to solve a Fuzzy Transportation problem based on extension principle. Lin introduced a genetic algorithm to solve Transportation with Fuzzy objective functions. Srinivasan [18] - [23] described the new methods to solve fuzzy transportation problem.

Nagoor Gani and Abdul Razak [13] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. A.Nagoor Gani, Edward Samuel and Anuradha [7] used Arshamkhan‟s Algorithm to solve a Fuzzy Transportation problem. Pandian and Natarajan [14] proposed a Fuzzy zero point method for finding a Fuzzy optimal solution for Fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers.

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2. PRELIMINARIES

In this section we define some basic definitions which will be used in this paper.

2.1 Definition – 1

The characteristic function $\mu_A(x)$ of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each

member in X. This function can be generalized to a function $\mu_{\tilde{A}}(x)$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu : X \to [0,1]$. The assigned value indicate the membership grade of the element in the set

A. The function $\mu_{\tilde{A}}(x)$ is called the membership function and the set $\tilde{A} = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1]\}$. is called a fuzzy set.

2.2 Definition – 2

A fuzzy set A , defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu : R \to [0,1]$ has the following characteristics

(i) A is normal. It means that there exists an x ∈ R such that $\mu(x) = 1$

(ii) A is convex. It means that for every $x_1, x_2 \in R$,

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}, \lambda \in [0,1]$$

(iii) μ_A is upper semi-continuous.

(iv) supp (A) is bounded in R.

2.3 Definition – 3

A fuzzy number A is said to be non-negative fuzzy number if and only if $\mu(x) = 0, \forall x < 0$

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2.4 Definition – 4

A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by, where a≤ b≤ c≤d.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a < x \leq b, \\ 1, & b < x < c, \\ \frac{d-x}{d-c}, & c \leq x < d, \\ 0, & x > d \end{cases}$$

2.5 Definition – 5

A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be non-negative (non - positive) trapezoidal fuzzy number. i.e. $\tilde{A} \geq 0 (\tilde{A} \leq 0)$ if and only if $a \geq 0 (c \leq 0)$. A trapezoidal fuzzy number is said to be positive (negative) trapezoidal fuzzy number i.e. $\tilde{A} > 0 (\tilde{A} < 0)$ if and only if $a > 0 (c < 0)$.

2.6 Definition – 6

Two trapezoidal fuzzy number $\tilde{A}_1 = (a, b, c, d)$ and $\tilde{A}_2 = (e, f, g, h)$ are said to be equal. i.e. $\tilde{A}_1 = \tilde{A}_2$ if and only if a=e,b=f,c=g,d=h.

2.7 Definition – 7

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Let $\tilde{A}_1 = (a, b, c, d)$ and $\tilde{A}_2 = (e, f, g, h)$ be two non-negative trapezoidal fuzzy number then

(i) $\tilde{A}_1 \oplus \tilde{A}_2$ = (a,b,c,d) ⊕ (e,f,g,h) = (a+e,b+f,c+g,d+h)

(ii) $\tilde{A}_1 \tilde{-} \tilde{A}_2$ = (a,b,c,d) − (e,f,g,h) = (a-h,b-g,c-f,d-e)

(iii) $\tilde{-}\tilde{A}_1$ = - (a,b,c,d) = (-d,-c,-b,-a)

(iv) $\tilde{A}_1 \otimes \tilde{A}_2$ = (a,b,c,d) ⊗ (e,f,g,h) = (ae,bf,cg,dh)

(v) $^1 \cong$ − − − − $_\alpha$ $\tilde{A}$ $(\frac{1}{d}, \frac{1}{c}, \frac{1}{b}, {}^1)$

2.8 Rou bast Ranking Technique

Rou bast ranking technique which satisfy compensation, linearity, and additivity properties and provides results which are consist of human intuition. If ã is a fuzzy number then the

Rou bast ranking is defined by $R(\tilde{a}) = \int_0^1 0.5(a_\alpha^L\ a_\alpha^U)d\alpha$, Where $(a_\alpha^L\ a_\alpha^U)$ is the α level cut of the fuzzy number $\tilde{a}$ and $(a_\alpha^L\ a_\alpha^U)$ = {((b-a)+a),(d-(d-c))}

In this paper we use this method for ranking the objective values. The Rou bast ranking index

$R(\tilde{a})$ gives the representative value of fuzzy number $\tilde{a}$.

Mathematical formulation of a fuzzy transportation problem

Mathematically a transportation problem can be stated as follows:

Minimize

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \qquad \text{------(1)}$$

Subject to

$$\left.\begin{array}{ll} \sum_{j=1}^{n} x_{ij} = a_i & j = 1,2,\ldots,n \\ \sum_{i=1}^{m} x_{ij} = b_j & i = 1,2,\ldots,m \\ x_{ij} \geq 0 & i = 1,2,\ldots,m,\ j = 1,2,\ldots,n \end{array}\right\} \text{----(2)}$$

Where c is the cost of transportation of an unit from the i^{th} source to the j^{th} destination, and the quantity x_{ij} is to be some positive integer or zero, which is to be transported from the

i^{th} origin to j^{th} destination. A obvious necessary and sufficient condition for the linear

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programming problem given in (1) to have a solution is that

$$\sum_{i=1} a_i = \sum_{j=1} b_j \quad \text{.......................} \quad (3)$$

(i.e) we assume that total availability is equal to the reequipment. If it is not true, a fictitious source or destination can be added. It should be noted that the problem has feasible solution if and only if the condition (2) satisfied. Now, the problem is to determine x_{ij}, in such a way that the total transportation cost is minimum

Mathematically a fuzzy transportation problem can be stated as follows: Minimize

$$z = \sum_{i=1}\sum_{j=1} c_{ij} x_{ij} \quad \text{.......................................} \quad (4)$$

Subject to

$$\sum_{j=1} \tilde{x}_{ij} = a_i \quad j = 1, 2, \ldots, n \quad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad i = 1, 2, \ldots, m,$$

$$x_{ij} \geq 0 \quad j = 1, 2, \ldots, n \qquad \text{---(5)}$$

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In which the transportation costs $\tilde{c}_{ij}$, supply $\tilde{a}_i$ and demand $\tilde{b}_j$ quantities are fuzzy quantities. An obvious necessary and sufficient condition for the fuzzy linear programming problem give in (4-5) to have a solution is that

$$\sum_{i=1}^{n} \tilde{b}_j \quad a_i = \sum^{m} \quad \ldots\ldots\ldots(6)$$

This problem can also be represented as follows:

	1	………	n	Supply
1	$\tilde{c}_{11}$	………	$\tilde{c}_{1n}$	a_1
. . . .	. . . .	. . . .	. . . .	. . . .
M	$\tilde{c}_{m1}$	……...	$\tilde{c}_{mn}$	a_m
Demand	$\tilde{b}_1$	………	$\tilde{b}_n$	

3. Alternate Method for Solving Transportation Problem

Following are the steps for solving Transportation Problem

Step – 1: From the given Transportation problem, convert fuzzy values to crisp values using ranking function.

Step – 2: Deduct the minimum cell cost from each of the cell cost of every row/column of the Transportation problem and place them on the right-top/right-bottom of corresponding cost.

Step – 3: Adding the cost of right-top and right – bottom and place the summation value in the corresponding cell cost.

Step – 4: Identify the minimum element in each row and column of the Transportation table and subtract on their corresponding the row and the column.

Step – 5: Find the sum of the values in the row and the column. Choose the maximum value and

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allocate the minimum of supply/demand in the minimum element of the row and column. Eliminate by deleting the columns or rows corresponding to where the supply or demand is satisfied.

Step – 6: Continue step -4 and step -5 until of all the supply and demand is met. Step – 7: Place the original transportation cost to satisfied cell cost.

Step – 8: Calculate the minimum cost.

That is,

$$\text{Total Cost} = \sum\sum C_{ij} X_{ij}$$

4. Numerical Example

Consider the Fuzzy Transportation Problem

	FD_1	FD_2	FD_3	FD_4	Fuzzy Capacity
FO_1	[1,2,3,4]	[1,3,4,6]	[9,11,12,14]	[5,7,8,11]	[1,6,7,12]
FO_2	[0,1,2,4]	[-1,0,1,2]	[5,6,7,8]	[0,1,2,3]	[0,1,2,3]
FO_3	[3,5,6,8]	[5,8,9,12]	[12,15,16,19]	[7,9,10,12]	[5,10,12,17]
Fuzzy Demand	[5,7,8,10]	[1,5,6,10]	[1,3,4,6]	[1,2,3,4]	

Solution

In Conformation to model the fuzzy transportation problem can be formulated in the following mathematical programming form

Min Z = R(1,2,3,4)x_{11} + R(1,3,4,6)x_{12} + R(9,11,12,14)x_{13} + R(5,7,8,11)x_{14} +R(0,1,2,4)x_{21}+R(-1,0,1,2)x_{22}+R(5,6,78)x_{23}+R(4,5,6,7)x_{24}+R(3,5,6,8)x_{31}+R(5,8,9,12)x_{32} +R(12,15,16,19)x_{33} + R(7,9,10,12)x_{34}

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^L \; a_\alpha^U)d\alpha$$

where (a_α^L, a_α^U) = {((b-a)+a),(d-(d-c))}

After applying ranking technique, we get

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Table -1

	D1	D2	D3	D4	Supply
O1	2.5	3.5	11.5	7.75	6.5
O2	1.75	0.5	6.5	1.5	1.5
O3	5.5	8.5	15.5	9.5	11
Demand	7.5	5.5	3.5	2.5	

Table – 2

	D1	D2	D3	D4	Supply
O1	2.5_{0}^{0}	3.5_{3}^{1}	11.5_{5}^{9}	$7.75_{6.25}^{5.25}$	6.5
O2	$1.75_{0.75}^{1.25}$	$.5_{0}^{0}$	6.5_{0}^{6}	1.5_{0}^{1}	1.5
O3	5.5_{3}^{0}	8.5_{8}^{3}	15.5_{9}^{10}	9.5_{8}^{4}	11
Demand	7.5	5.5	3.5	2.5	

Table – 3

	D1	D2	D3	D4	Supply
O1	0	4	14	11.5	6.5
O2	2	0	6	1	1.5
O3	3	11	19	12	11
Demand	7.5	5.5	3.5	2.5	

With the help of the method, we get

Table – 4

	D1	D2	D3	D4	Supply
O1	0	5.5 4	1 14	11.5	6.5
O2	2	0	1.5 6	1	1.5
O3	7.5 3	11	1 19	2.5 12	11
Demand	7.5	5.5	3.5	2.5	

Finally, we get

Table – 5

	D1	D2	D3	D4	Supply
O1		**5.5** **3.5**	**1** **11.5**		6.5
O2			**1.5** **6.5**		1.5
O3	**7.5** **5.5**		**1** **15.5**	**2.5** **9.5**	11
Demand	7.5	5.5	3.5	2.5	

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Hence (4+3-1)=6 cells are allocated and hence we got our feasible so ln next we calculate total cost and its corresponding allocated value of supply and demand

Total Cost (5.5X7.5) + (3.5X5.5) + (11.5X1) + (6.5X1.5) + (15.5X1) + (9.5X2.5) = 121

This is a basic feasible solution. The solution obtained using NCM, LCM, VAM and MODI/Stepping stone methods respectively. Hence the basic feasible solution obtained from method is optional soln.

5. Conclusion

In this paper, the transportation costs are considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy transportation problem of trapezoidal fuzzy numbers has been transformed into crisp transportation problem using robust ranking technique indices. Numerical examples show that by this method we can have the optimal solution as well as the crisp and fuzzy optimal total cost. By using Rou bast ranking method we have shown that the total cost obtained is optimal. Hence, this will be helpful for decision makers who are handling logistic and supply chain problems in fuzzy environment. For future research we propose effective implementation of the trapezoidal fuzzy numbers in all fuzzy problems.

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Reliability Measures of Hybrid Electric Vehicles (HEVs) and Plug in Hybrid Electric Vehicles (PHEVs)

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Abstract

This study deals with the analysis of availability and reliability of hybrid electric vehicles (system 1) and plug-in hybrid electric vehicles (system 2). The purpose of this study is to find out the opinion of consumers who can afford their own hybrid car. The distribution of failure and repair rates is assumed to be exponential. A method of linear differential equations (LDE) is used to estimate reliability metrics such as average system failure and steady-state availability. Some special cases were evaluated using different values of the failure rates. In addition, we examined how the failure rate affected the system performances measures and we demonstrated the basic involved concept comparing the results of both systems. The results are also presented graphically using MATLAB software.

Keywords: *Reliability, Steady -State Availability, Linear Differential Equation, Mean Time to System Failure(MTSF).*

1. Introduction

In the subject of system reliability, a number of authors analyzed and assessed reliability matrices, including MTSF, steady-state availability, busy period of repairmen, and cost analysis with the Markov renewal process using regeneration point approach. In this work, dependability measures are assessed using linear differential equations technique. This approach is less complicated than the others, and MATLAB software can be used to carry out computations.Many authors previously employed linear differential equations techniques to evaluate reliability measures for various systems. The availability analysis and reliability measure of two non-identical systems were examined by El-Said et al. using the linear differential equation [1].El.sherbeny at analysed the behaviour of some industrial systems in light of the cost-free warranty policy [2].Gupta and Mittal investigated the stochastic behaviour of a two-unit warm standby system with two types of repairmen and varying levels of patience time [3]. Mokaddies et al. assessed the reliability and availability of two dissimilar-unit cold standby systems with three modes using a linear differential equation with no cost-benefit analysis [4]. Gao at el Studied a *K*-out-of M+W+C: G mixed standby system with an unreliable repair facility [5]. Uba Ahmad Ali et al. evaluated the dependability of a two different unit cold standby system with three modes using the Kolmogorov Forward equation approach [6]. Yusuf, I.et. al. studied Stochastic Modeling and performance measures of redundant system operating in different conditions[9]. Pradeep K. Joshi et al discussed the dependability and availability of a two-unit standby redundancy system using the linear differential

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equation solution technique [7].Lane et. al. analysed data from a survey of drivers (n = 1080) administered in late 2013 to assess factors that influence potential car buyers to consider two different types of plug-in electric vehicles (PEVs) in the United States: plug-in hybrid electric vehicles (PHEVs) and battery electric vehicles (BEVs) [8].The goal of this study is to investigate the dependability matrices, such as MTSF and steady state availability analysis, of a hybrid four-wheeler (system 1) and plug-in hybrid four-wheeler (system 2) using linear differential equation technique. Plug-in hybrid four-wheeler (System 2) is superior to hybrid four-wheeler(System 1) based on the computation presented in the study regarding the influence of the battery charging option and switching.A graphical representation of measures of system effectiveness of both the system is also explored.

2. Model Description and Assumptions

In this work, two types of electrical vehicles were studied: hybrid electric vehicles (system 1) and plug-in hybrid electric vehicles (system 2).

2.1 **Hybrid Electric Vehicle:** Hybrid electric vehicles are powered by an internal combustion engine and an electric motor which uses energy stored in batteries.A hybrid electric vehicle cannot be plugged in to charge the battery.

2.2 **Plug in Hybrid Electric Vehicle**: Plug in hybrid electric vehicle use batteries to power an electric motor, as well as another fuel such as gasoline or diesel to power an internal combustion engine or other propulsion sources. PHEV can charge their batteries through charging equipment and regenerative braking.

Throughout the study of research paper, the following assumptions has been made:

- System 1 (HEV) can generate electricity through regenerative braking rather than by plugging into a charging station to recharge the vehicle's battery.
- The hybrid four-wheeler in system 1 continues to run even if the battery failed.
- System 2 (PHEV) will only use its internal combustion engine as a backup and will be primarily driven by an electric motor.
- The switch in system 2 (PHEV) is utilized to turn on the petrol supply.
- System 2 (PHEV) uses an automatic switch to start the engine immediately when the battery dies (fails), provided the switch is in working order at the time of need; otherwise, the engine won't start until the switch is repaired.
- System 1 has only two modes, namely failure and normal.
- System 2 has three operating modes: normal, partial, and failure.
- Repair is flawless (as good as new)
- Only one change may be made at a time in a single state.
- All failure rates and repair rates are constant.

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- Failure and repair rates are followed by an exponential distribution.

3 Notation and Symbol

S_i : Transition state of the system , i =0,1 , 2 ,3 ,4

P_N –Petrol supply Normal

B_N –Battery fully charged

B_{NP} –Battery partially charged

P_{NP} –Petrol supply is partially

B_F-Battery failed.

P_F-Petrol supply failed.

α –failure rate of petrol supply

β - failure rate of battery

α' -failure rate of petrol when battery already failed

β' -failure rate of battery when petrol already failed.

δ-Repair rate of Petrol supply

λ-Repair rate of battery

δ_1-rate of charging battery

α_2-rate of completion of battery charging

α_1-rate of filling petrol

μ_1-rate of completion of filling petrol

θ-replacement rate of both petrol &battery

4 Transition probability of Hybrid Electric Vehicle (System 1)

Figure 1 shows the transition probability of different states of system 1.

Up State ;$S_0 \equiv (B_N, P_N)$, $S_1 \equiv (B_N, P_F)$, $S_2 \equiv (B_F, P_N)$

Down State ; $S_3 \equiv (B_F, P_F)$

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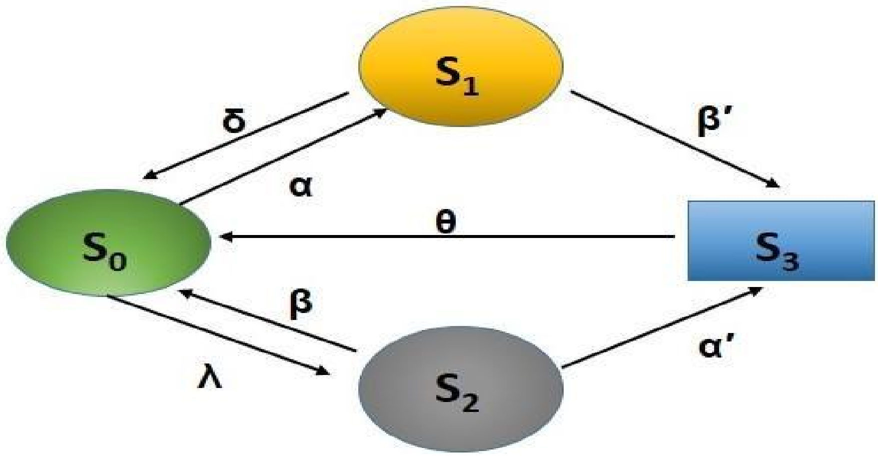

Figure 1. State Transition Diagram for the system1

5 Measures of System Effectiveness of System 1

5.1 Mean time to system failure (MTSF).

By applying linear differential equation technique and above assumptions, the mean time to system failure (MTSF) of the proposed system is determined. Define Pi(t) as the probability that the system is in state Si at time t, based on Figure 1. Let P(t) represent the probability row vector at time t. Consider the inditial conditions as :

$P(0)= [\, P_0(0), P_1(0), P_2(0), P_3(0)],\ = [\, 1, 0\ , 0\ ,0\ ]$ (1) The derived system of differential equations is as follows :

$(dp_0(t))/dt$ = -(α+β) $P_0(t)$+ δ $P_1(t)$+ λ $P_2(t)$+ θP_3(t)

$(dp_1(t))/dt$= -(β^'+ δ)P_1(t)+ α $P_0(t)$

$(dp_2(t))/dt$= -(α^' + λ)P_2(t) + βP_0(t)

$(dp_3(t))/dt$= - θ P_3(t) +β^' P_1(t) +α^' $P_2(t)$

which can be expressed in matrix form as

$$\frac{dp(t)}{dt} = A\,P \qquad (2)$$

where

$$A = [-\ (\alpha\ +\ \beta)\ \delta\ \lambda\ \theta\ \alpha\ \ -\left(\beta^{'}\ +\ \delta\right)\ 0\ 0\ \beta\ 0\ \ -\left(\alpha^{'}\ +\ \lambda\right)\ 0\ 0\ \beta^{'}\ \alpha^{'}\ \ -\ \theta\ ]$$

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We eliminate the rows and columns of the absorption state of the matrix A and transpose it to create a new matrix called Q because evaluating the transition solution is difficult. The expected time to reach an absorbing state is determined from

$$E[T_{P(0)\rightarrow(absorbing)}] = P(0)\int_0^{\infty} e^{Qt}dt$$

and

$$\int_0^{\infty} e^{Qt}dt = -Q^{-1}, \text{since } Q^{-1} < 0$$

where

$$Q = [-(\alpha+\beta)\ \alpha\ \beta\ \delta\ -(\beta'+\delta)\ 0\ \lambda\ 0\ -(\alpha'+\lambda)]$$

The MTSF can be expressed explicitly as:

$$MTSF1 = E[TP(0)\rightarrow(absorbing)] = P(0)(-Q^{-1})[1\ 1\ 1] \tag{3}$$

$$MTSF_1 = \frac{(\alpha+\beta_1')(\lambda+\alpha')+\delta(\lambda+\alpha')+\beta(\delta+\beta')}{\alpha\beta'(\lambda+\alpha')+\beta\alpha'(\delta+\beta')} \tag{4}$$

5.2 Steady -State Availability Analysis of the System 1

The initial condition for the availability analysis in Figure 1 is the same as for the reliability case.

$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0))] = [1, 0, 0, 0]$

The systemof differential equations can be expressed as:

$\dot{P} = A\,P$

$$[p'_0\ p'_1\ p'_2\ p'_3] = [-(\alpha+\beta)\ \delta\ \lambda\ \theta\ \alpha\ -(\beta'+\delta)\ 0\ 0\ \beta\ 0\ -(\alpha'+\lambda)\ 0\ 0\ \beta'\ \alpha'\ \theta][P_0\ p_1\ p_2\ P_3]$$

The steady-state availability is given by

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$A_{T_1}(\infty) = 1 - P_3(\infty)$ (5)

In the steady - state availability,the derivatives of the state probabilities become zero so that

$A\ P(\infty) = 0$ *(6)*

which is in matrix form

$$\left[- (\alpha + \beta)\ \delta\ \lambda\ \theta\ \alpha\ - \left(\beta^{'} + \delta\right)\ 0\ 0\ \beta\ 0\ - \left(\alpha^{'} + \alpha\right)\ 0\ 0\ \beta^{'}\ \alpha^{'}\ \theta\ \right]\left[P_0(\infty)\ P_1(\infty)\ P_2(\infty)\ P_3(\infty)\ \right] = [0\ 0\ 0\ 0\ ]$$

(7)

$$P_0(\infty) + p_1(\infty) + P_2(\infty) + p_3(\infty) = 1 \qquad (8)$$

To get $P_3(\infty)$ we substitute (8) in one of the redundant rows of (7) and use MATLAB to obtain the solution of the following system of linear equations in matrix form

$$\left[- (\alpha + \beta)\ \delta\ \lambda\ \theta\ \alpha\ - \left(\beta^{'} + \delta\right)\ 0\ 0\ \beta\ 0\ - (\alpha + 1)\ 0\ 1\ 1\ 1\ 1\ \right]\left[P_0(\infty)\ P_1(\infty)\ P_2(\infty)\ P_3(\infty)\ \right] = [0\ 0\ 0\ 1\ ]$$

(9)

The solution of (9) provides the steady - state availability for Figure 1. The explicit expression for $AT_1(\infty)$is :

$$A_{T_1}(\infty) = \frac{\alpha\left(\alpha^{'}+\lambda\right)\left(\beta^{'}+\theta\right)+\left(\beta^{'}+\delta\right)\left(\alpha^{'}\beta+\theta\lambda\right)+\theta\left(\alpha^{'}\beta^{'}+\alpha\delta\right)}{\alpha\left(\alpha^{'}+\lambda\right)\left(\beta^{'}+\theta\right)+\left(\beta^{'}+\delta\right)\left[\beta\left(\alpha^{'}+\theta\right)+\theta\left(\alpha^{'}+\lambda\right)\right]} \qquad (10)$$

6. Transition probability of Plug- in hybrid electric vehicle(System 2)

Figure2 shows the transition probability of different states of system 2.

Up State ; $S_0 \equiv (B_N,P_N)$, $S_1 \equiv (B_{NP},P_N)$, $S_2 \equiv (B_N,P_{NP})$, $S_3 \equiv (B_N,P_F)$, $S_4 \equiv (B_F,P_N)$

Down State $S_5 \equiv \left(B_F, P_F\right)$

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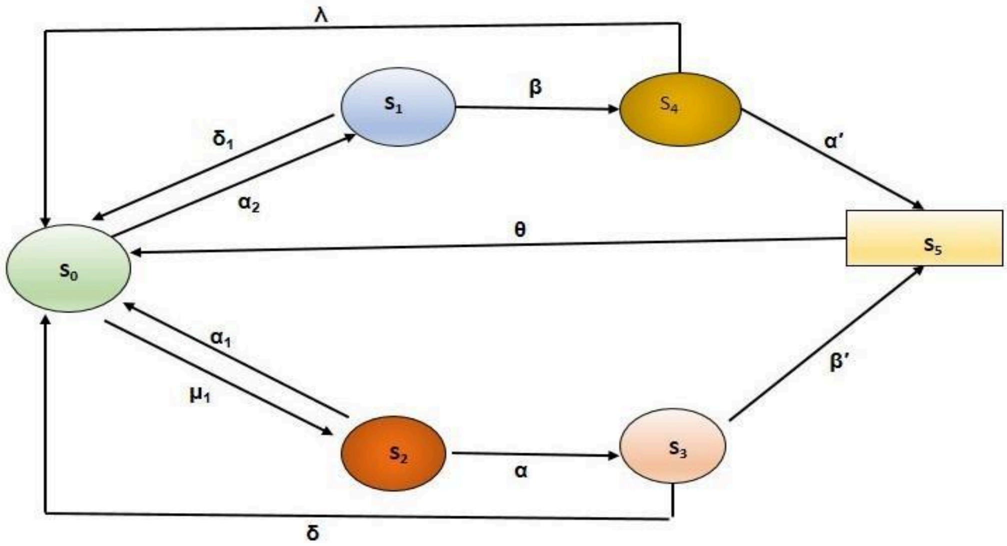

Figure 2; State transition diagram for the Second system

7 Measures of System Effectiveness of System 2

7.1 Mean Time to System Failure

By applying the linear differential equation technique and the above assumptions, the mean time to system failure (MTSF) of the proposed system is determined. Define Pi(t) as the probability that the system is in state Si at time t, based on Figure 2. Let P(t) represent the probability row vector at time t. Consider the inditial conditions as :

By applying the linear differential equation technique and the aforementioned assumptions, the mean time to system failure (MTSF) of the suggested system is determined. Define Pi(t) as the probability that the system will be in state Si at time t based on Figure 2. Let P(t) represent the probability row vector at time t. Consider the inditial conditions as :

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)] = [1, 0, 0, 0, 0, 0] \quad (11)$$

The derived system of differential equations is as follows:

$$\frac{dp_0(t)}{dt} = -(\alpha_2 + \alpha_1)\, P_0(t) + \delta_1 P_1(t) + \mu_1 P_2(t) + \delta P_3(t) + \lambda P_4(t) + \theta P_5(t)$$

$$\frac{dp_1(t)}{dt} = -(\beta + \delta_1) p_1(t) + \alpha_2 P_0(t)$$

$$\frac{dP_2(t)}{dt} = -(\alpha + \mu_1) P_2(t) + \alpha_1 P_0(t)$$

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$$\frac{dP_3(t)}{dt} = -\left(\beta^{'} + \delta\right)P_3(t) + \alpha P_2(t)$$

$$\frac{dP_4(t)}{dt} = -\left(\lambda + \alpha^{'}\right)P_4(t) + \beta P_1(t)$$

$$\frac{dP_5(t)}{dt} = -\theta P_5(t) + \beta^{'} P_3(t) + \alpha^{'} P_4(t)$$

which can be expressed in matrix form as

$$\dot{P} = A\,P \qquad (12)$$

Where

A

$$= \left[-\left(\alpha_2 + \alpha_1\right)\, \delta_1\, \mu_1\, \delta\, \lambda\, 0\, \alpha_2\ -\left(\beta + \delta_1\right)\, 0\, 0\, 0\, 0\, \alpha_1\, 0\ -\left(\alpha + \mu_1\right)\, 0\, 0\, 0\, 0\, 0\, \alpha\ -\left(\beta^{'} + \delta\right)\, 0\, 0\, 0\, \beta\, 0\, 0$$

We eliminate the rows and columns of the absorption state of the matrix A and transpose it to create a new matrix called Q because evaluating the transition solution is difficult. The expected time to reach an absorbing state is determined from

$$E[T_{P(0)\rightarrow(absorbing)}] = P(0)\int_0^{\infty} e^{Qt}dt$$

and

$$\int_0^{\infty} e^{Qt}dt = -Q^{-1}, \text{since } Q^{-1} < 0$$

where

Q

$$= \left[-\left(\alpha_2 + \alpha_1\right)\, \alpha_2\, \alpha_1\, 0\, 0\, \delta_1\ -\left(\beta + \delta_1\right)\, 0\, 0\, \beta\, \mu_1\, 0\ -\left(\alpha + \mu_1\right)\, \alpha\, 0\, \delta\, 0\, 0\ -\left(\beta^{'} + \delta\right)\, 0\, \lambda\, 0\, 0\, 0\ -\left(\alpha^{'} + \lambda\right) \right]$$

$$MTSF_2 = E[T_{P(0)\rightarrow(absorbing)}] = P(0)(-Q^{-1})[1\ 1\ 1\ 1\ 1\ ] \qquad (13)$$

$$MTSF_2 = \frac{\alpha_2\beta\alpha^{'}\left(\alpha+\mu_1\right)\left(\delta+\alpha^{'}\right)+\alpha_1\alpha\beta^{'}\left(\lambda+\alpha^{'}\right)\left(\beta+\delta_1\right)}{\left(\lambda+\alpha^{'}\right)\left[\left(\alpha+\mu_1+\alpha_1\right)\left(\beta+\delta_1\right)\left(\delta+\beta^{'}\right)+\alpha_1\alpha\left(\beta+s_1\right)\right]+\alpha_2\left(\delta+\beta^{'}\right)\left(\alpha+\mu_1\right)\left(\lambda+\alpha^{'}+\beta\right)} \qquad (14)$$

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7.2 Steady **-State Availability Analysis of the System**

The initial condition for the availability analysis in Figure 2 is the same as for the reliability case.

$$P(0) = \left[P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)\right] = [1, 0, 0, 0, 0, 0]$$

The system of differential equation can be expressed as

$$\left[p^{\cdot}_0\, p^{\cdot}_1\, p^{\cdot}_2\, p^{\cdot}_3\, p^{\cdot}_4\, p^{\cdot}_5\right] =$$

$$\left[-\left(\alpha_2+\alpha_1\right) \delta_1\, \mu_1\, \delta\, \lambda\, 0\, \alpha_2\, -\left(\beta+\delta_1\right) 0\,0\,0\,0\, \alpha_1\, 0\, -\left(\alpha+\mu_1\right) 0\,0\,0\,0\,0\, \alpha\, -\left(\beta^{'}+\delta\right) 0\,0\,0\, \beta\, 0\,0\, -\left(\alpha^{'}+\lambda\right.\right.$$

The steady-state availability is given by

$$A_{T_2}(\infty) = 1 - P_5(\infty) \quad (15)$$

In the steady - state availability, the derivatives of the state probabilities become zero so that

$$A\,P(\infty) = 0$$

which in matrix form

$$\left[-\left(\alpha_2+\alpha_1\right) \delta_1\, \mu_1\, \delta\, \lambda\, 0\, \alpha_2\, -\left(\beta+\delta_1\right) 0\,0\,0\,0\, \alpha_1\, 0\, -\left(\alpha+\mu_1\right) 0\,0\,0\,0\,0\, \alpha\, -\left(\beta^{'}+s\right) 0\,0\,0\, \beta\, 0\,0\, -\left(\alpha^{'}+\lambda\right.\right.$$

=[0 0 0 0 0 0]   (16)

$$P_0(\infty) + p_1(\infty) + P_2(\infty) + p_3(\infty) + p_4(\infty) + p_5(\infty) = 1 \qquad (17)$$

To get$P_5(\infty)$ we substitute (17) in one of the redundant rows of (16) and use MATLAB to obtain the solution of the following system of linear equations in matrix form.

$$\left[-\left(\alpha_2+\alpha_1\right) s_1\, \mu_1\, \delta\, \lambda\, 0\, \alpha_2\, -\left(\beta+\delta_1\right) 0\,0\,0\,0\, \alpha_1\, 0\, -\left(\alpha_1\right) 0\,0\,0\,0\,0\, \alpha\, -\left(\beta^{'}+\beta\right) 0\,0\,0\, \beta\, 0\,0\, -\left(\alpha^{'}+\lambda\right) 0\, 1\right.$$

=[0 0 0 0 0 1]   (18)

The solution of (18) provides the steady state probabilities in the availability case for Figure 2.The explicit expression for $AT_2(\infty)$ is :

$$\theta\left(\lambda+\alpha^{'}\right)\left(\alpha+\mu_1\right)\left(\beta+\delta_1\right)\left(\delta+\beta^{'}\right) + \alpha_2\theta\left(\alpha+\mu_1\right)\left(\delta+\beta^{'}\right)\left(\lambda+\alpha^{'}+\beta\right)\ AT_2(\infty) = \frac{+\alpha}{2\beta\left(\delta+\beta^{'}\right)\left(\theta+\alpha\right.}$$

(19)

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8. Result and Discussion

We plot the MTSF and Steady State Availability for each model versus α and β, respectively, while keeping the other parameters fixed at $\alpha' = 1.5, \beta' = 1.5$, $\delta = 2.5$, $\lambda = 2$, $\mu_1 = \alpha_2 = 2.5$, $\delta_1 = \alpha_1 = 3$, $\theta = 3.5$ in order to observe the behavior of the system.We take β = 0.4 for the curve against α, and we take α = 0.4 in addition to other parameters for the curve against β.

Table 1 The relationship between first and second system's availability, MTSF, and failure rate "α"

α	$MTSF_1$	$MTSF_2$	$AT_1(\infty)$	$AT_2(\infty)$
0.1	5.44	17.72	0.904	0.984
0.2	4.72	14.26	0.907	0.980
0.3	4.18	12.05	0.910	0.976
0.4	3.78	10.51	0.912	0.973
0.5	3.45	9.39	0.915	0.970
0.6	3.18	8.53	0.917	0.967
0.7	2.97	7.84	0.918	0.965
0.8	2.79	7.30	0.921	0.962
0.9	2.63	6.84	0.923	0.960
1.0	2.50	6.46	0.924	0.957

Table 2: The relationship between the first and second system's availability, MTSF, and failure rate "β"

β	$MTSF_1$	$MTSF_2$	$AT_1(\infty)$	$AT_2(\infty)$
0.1	5.85	46.8	0.975	0.982
0.2	4.90	29.7	0.953	0.979
0.3	4.25	22.11	0.932	0.976
0.4	3.78	17.81	0.913	0.973
0.5	3.41	15.05	0.894	0.971
0.6	3.12	13.12	0.870	0.968
0.7	2.89	11.83	0.860	0.966
0.8	2.70	10.61	0.840	0.964
0.9	2.53	9.75	0.830	0.962
1.0	2.40	9.01	0.820	0.960

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Figure 3;MTSF against α

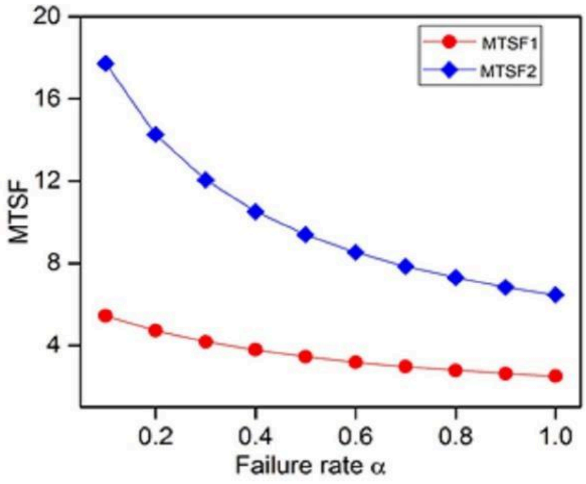


Figure 4; Availability against α

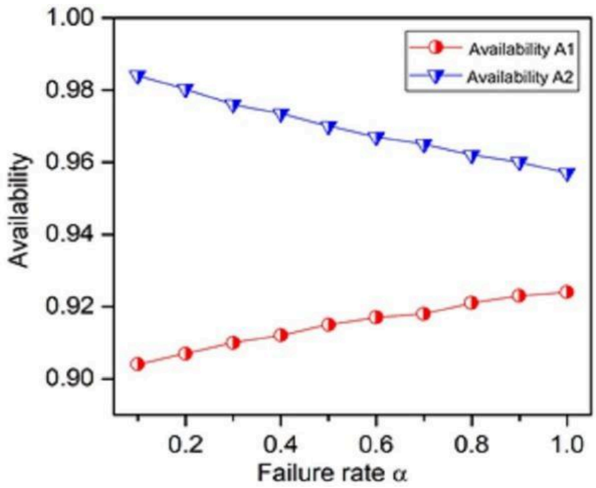


Figure 5; MTSF against β

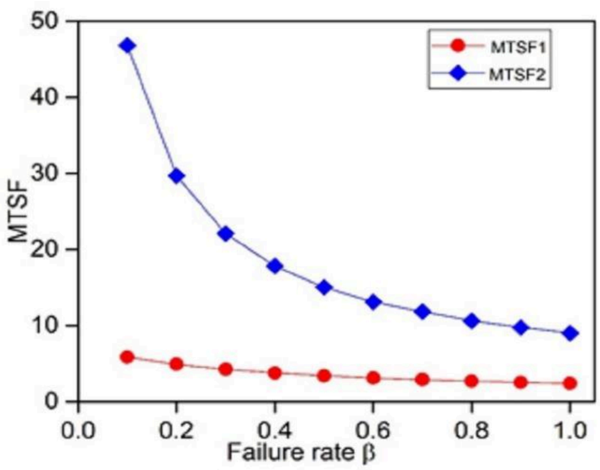
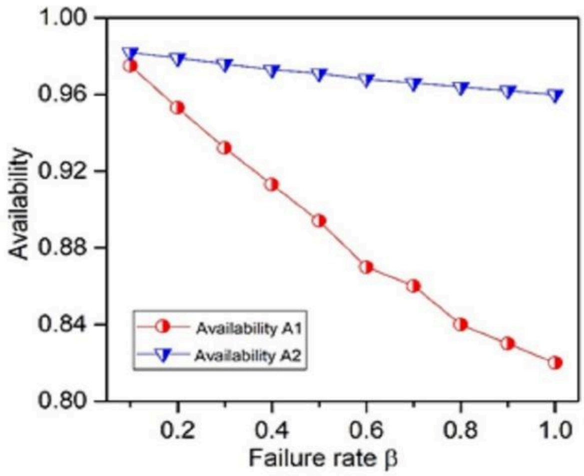


Figure 6; availability against β



As the value of α increases, the MTSF and availability of both systems decline, whereas the availability of system 1 marginally increases, as seen in figures 3 and 4. The graph makes it evident that system 2 has a higher mean time to system failure and availability than system 1.

The mean time to system failure and availability results for the two systems under study are plotted against the failure rate β in Figures 5 and 6.The figure clearly shows that, in comparison to system 1, system 2 has a higher mean time to system failure and availability.

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Conclusion

This work uses Linear Differential Equation Techniques to analyze the availability and reliability of suggested systems, namely Plug-in Hybrid Electrical Vehicles (System 2) and Hybrid Electrical Vehicles (System 2). The impact of failure rate on both system's MTSF and steady state availability is also monitored in order to observe system behavior. From Figures 3 to 6, we can conclude that plug-in hybrid electrical vehicles (System 2) with switching and battery charging options have higher MTSF and availability than Hybrid Electric Vehicles (System 1), which do not have battery charging options. Therefore, Plug-in Hybrid Electrical Vehicles are preferable to hybrid electric vehicles.

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QUEUING FOR SUCCESS : A QUICK LOOK AT SERVICE OPTIMIZATION

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ABSTRACT

This comprehensive review explores the application of queuing theory in optimizing service systems across diverse sectors. The studies analyzed delve into the intricacies of minimizing customer wait times and maximizing server utilization. Spanning from Fair Price Shops, banks, and post offices to supermarkets, healthcare centers, and petrol stations, the research employs various queuing models such as M/M/C, GI/M/c, and GI/M/1/N. Methodologies encompass case studies, simulation modeling, and mathematical analysis, providing insights into factors influencing service efficiency and customer satisfaction. Findings underscore the versatility and effectiveness of queuing theory, suggesting avenues for future research, including advanced queuing models, real-time analytics, and the integration of emerging technologies. Practical implementation in real-world service environments remains crucial for continuous improvement.

Keywords: Queuing theory, service systems, queuing models, optimization, customer satisfaction, waiting times, server utilization, simulation modeling, communication networks, healthcare services, banking, customer service, queuing systems, queue management.

INTRODUCTION

Queuing theory, a fundamental paradigm in operations research, plays a crucial role in understanding and enhancing the performance of service systems across various sectors. This article presents a comprehensive review of studies applying queueing theory to optimize service systems, with a focus on minimizing customer wait times and maximizing server utilization. The diverse applications range from Fair Price Shops (FPS) and banks to supermarkets, healthcare centers, and petrol stations. Each study explores specific aspects, such as arrival rates, waiting times, and server configurations, contributing valuable insights to the field of service system management.

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OBJECTIVE

To provide a comprehensive overview of the existing literature on queuing theory, focusing on its diverse applications in service systems, and to instill key insights contributing to optima

RESEARCH METHODOLOGY

The reviewed studies employ diverse research methodologies, including case studies, simulation modeling, mathematical modeling, and data analysis. Researchers utilize queuing models such as M/M/C, GI/M/c, and GI/M/1/N to analyze and optimize service systems. Simulation modeling is prevalent, particularly in studies assessing the optimal number of servers in specific contexts. The research also delves into factors influencing service efficiency, customer satisfaction, and the impact of various strategies on queue management zing service efficiency and customer satisfaction across different sectors

REVIEW OF LITERATURE

In a recent study conducted by Sasi, Subramanian, &Ravichandran , the research explores the application of queueing theory in Fair Price Shops (FPS), government buildings, banks, post offices, and other service sectors. The objective of this study is to achieve an optimal equilibrium between minimising wait times and maximising server utilisation by taking into account several aspects such as server utilisation, arrival rate, and service rate. The primary emphasis of this research is the implementation of the M/M/C queuing model in order to optimise queues at FPS (first-person shooter) venues. Simulation modelling is employed to ascertain the most favourable quantity of servers required to get a desired frame rate (FPS) in the region of Kerala, India. This study investigates the effects of many factors, including arrival rate, waiting time, and server utilisation.

The study conducted by Anand and Arora 1 aimed to assess the efficacy of customer service and waiting periods in Indian banks using a case study. The objective of their study was to ascertain the variables that influence the efficiency of customer service and to provide an estimation of the duration customers had to wait. The research conducted in this study was to identify and analyse potential bottlenecks in service delivery inside Indian banks, with the ultimate goal of providing recommendations to improve customer service.

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The study conducted by Bhardwaj et al. 2 examined a voice packetized statistical multiplexing system through the application of a fuzzy queuing model. The research conducted by the authors focused on the optimisation of communication networks, with particular emphasis on cases where voice traffic is predominant.

In their study, Kiataramkul and Neamprem 3 conducted an investigation of the effectiveness of a queuing model that incorporates many servers. The specific context of their analysis was centred around bank token systems and the impact on client waiting times. The research investigated several configurations of queueing theory in order to compute variables relevant to service.

In their study, Onoja et al. 4 proposed a mathematical model based on many servers and an exponentially distributed framework. This model aimed to analyse and predict various factors such as client waiting times, service rates, and arrival rates inside a banking environment. The objective of their study was to enhance the allocation of resources and minimise the duration customers spend waiting.

A study was undertaken by Jhala, Bhathawala, and Gujarat 5 with the objective of examining the utilisation of queueing theory within the context of supermarkets. The objective of their analysis was to maximise the efficiency of queue management for clients waiting outside the establishment. The study conducted a comparison between single queue and multiple queue multiserver systems, revealing the benefits of employing a single queue, multi-server method in terms of minimising client wait times and related expenses.

Thangaraj and Rajendran 6 conducted a study that investigated a queueing system characterised by batch arrivals and two distinct service patterns. The model takes into account scenarios in which the server would offer bulk service if the queue length surpasses a specific threshold 'a' following a period of inactivity, while otherwise providing single service. The investigation computed the distributions of queue sizes and assessed multiple performance indicators while considering specific situations within the model.

The objective of the study conducted by Agyei, Asare-Darko, and Odilon 7 was to provide guidance to bank management in optimising the staffing of tellers by identifying the optimal balance between minimising total economic costs (comprising waiting cost and service cost) and reducing customer waiting times. The researchers employed data collected from the Kumasi Main Branch of Ghana Commercial Bank Ltd in order to construct a queuing model with many servers operating in a single line. The findings of the analysis indicate that a five-teller system demonstrated superior performance compared to both four and six-teller systems in terms of average waiting times and overall economic expenses. These results suggest that implementing a five-teller system could be a cost-effective measure to improve customer satisfaction.

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In their study, Harley et al. 8 examined the effects of several customer service components implemented by Nigerian banks on the financial performance of these institutions. A significant relationship was discovered between the mean duration of client waiting in queues and the financial viability of banks in Nigeria, so suggesting that proficient queue management and service provision play a crucial role in determining a bank's level of achievement.

Chandra and Madhu 9 conducted a study that investigated a Markovian queuing system characterised by the presence of multiple service counters and finite waiting times. In this particular concept, a pair of servers were employed at separate counters in order to deliver a comprehensive level of service to an individual customer. The primary objective of the analysis was to ascertain the distribution of queue sizes in a condition of equilibrium, while also examining the consequences of modifying specific factors. The research emphasised the possibility of achieving more accurate results by integrating state-dependent rates into the multi-counter system model. Moreover, it is underscored that the modelling of queueing systems with blocking is of utmost importance. However, it also highlights the necessity to shift focus towards the factors of bulk arrivals and service.

In a study conducted by Kamau 10, the focus was on examining the relationship between waiting queue management and customer satisfaction in commercial banks in Kenya. The objective of Kamau's study was to examine the efficacy of Kenyan banks in addressing customer grievances pertaining to extended waiting periods. This study investigated the strategies employed by commercial banks in the management of waiting lines, the obstacles they face in implementing these strategies, and the consequent effects on customer satisfaction. This research enhances the comprehension of queueing management in service systems, particularly in the setting of commercial banks in Kenya.

The study conducted by Ohaneme et al. 10 focuses on the evaluation of queuing systems at petrol stations. The present study employed petrol stations as a case study to assess the importance of queuing systems in service operations. The researchers noted that petrol stations exhibit a tendency to service consumers in a random manner, resulting in the formation of lengthy queues and prolonged waiting durations. The implementation of the M/M/6 queuing system has been demonstrated to yield substantial improvements in the efficiency of client services when rigorously applied. This study offers valuable insights into the optimisation of queueing systems within service sectors.

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In their study titled "Single Working Vacation in GI/M/1/N and GI/M/1/∞," Banik, Gupta, and Pathak 12 investigate the use of single working vacation policies in the context of GI/M/1/N and GI/M/1/∞ systems. The topic of interest is queueing systems. They. conducted a study aimed at assessing the effects of a solitary working vacation on queueing systems. The authors employed embedded Markov chain and additional variable techniques to estimate queue length distributions and other essential performance metrics. This study aims to enhance comprehension of the impact of server vacations on the performance of queueing systems.

The study conducted by Rao et al. 13 focuses on the use of queuing theory in the context of communication networks. The present study has made substantial progress in the utilisation of queuing theory within the realm of communication networks. The research examined the arrival and broadcasting procedures occurring at various network nodes, constructing a comprehensive model for an interdependent communication network. The results of this study have significant significance for the enhancement of network architecture and administration, specifically in relation to optimising data flow inside communication networks.

The field of queueing theory is of paramount importance in comprehending and enhancing the performance of service systems. The research undertaken by Jacob and Szyszkowski 14 centred on the analysis of call centre data. The researchers noted that the duration of desertion in call centres adheres to a universally applicable and autonomous probability distribution. Based on their empirical findings, the researchers determined that the Poisson distribution exhibited the highest level of suitability as a model for call centres, specifically in relation to service times.

In their study, Adeleke 15 focused on university health centres and utilised a single-server queuing model to estimate waiting times. The researchers' model, which made the assumption of Poisson arrival with exponential service rates and implemented the First-In-First-Out (FIFO) queue discipline, facilitated the estimation of patient arrivals and waiting durations in emergency departments. These models possess significant value in augmenting the efficiency of healthcare systems.

In their study, Cochran and Roche 16 conducted an investigation of a range of modelling strategies aimed at mitigating the issues of hospital bed shortages and congestion. The scope of their study included the utilisation of empirical equations, including linear and nonlinear equations, as well as the application of time series forecasting and queuing theory-based models. It is worth noting that models based on queuing theory demonstrated superior performance compared to techniques based on formulas. These queuing theory-based

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models provided more effective strategies for optimising the distribution of beds and enhancing the quality of healthcare services.

Queueing theory is a fundamental paradigm that provides insights into the dynamics of waiting lines and service systems. The efficiency of queuing in traditional and modern banks in Nigeria was assessed by a comparative analysis undertaken by Kasum et al. in 2006. The collection of primary data was conducted by utilising an inverted-funnel questionnaire that was administered by the bank clients themselves. The results of the study revealed that consumers of contemporary banks encountered notably reduced waiting durations in comparison to customers of conventional banks, thereby emphasising the significance of effective service provision within the banking industry.

In their study, Pei-Chun et al. 17 utilised queueing theory as a framework to assess the efficacy of different Taiwanese banking institutions, including postal banking services. The research conducted by the authors centred on the examination of several operations of automated teller machines (ATMs), including cash withdrawal, fund transfer, password reset, and balance inquiry. The effectiveness of ATM services was evaluated through the utilisation of a queueing model, which revealed the necessity of augmenting the number of ATMs in certain financial institutions in order to mitigate consumer wait times.

The researchers Green et al. 17 employed the M/M/s queuing model in their study to investigate the interplay between service delays, patient utilisation, and the optimal number of servers necessary for the functioning of healthcare systems. The aforementioned findings make a valuable contribution to the existing body of knowledge about the optimisation of healthcare services.

The study conducted by Tian and Zhang 20 examined a queueing system that incorporated several servers and a vacation policy with a (d, N)-threshold. This policy permitted a designated quantity of inactive servers to concurrently engage in vacation periods. The primary aim of their study was to determine the ideal values for the variables d and N. The research conducted by Ke et al. (2009) expanded upon previous studies by investigating the optimal vacation approach (d, c) for an M/M/c/N queue with servers that are prone to failures and require repair processes. This study contributed to the advancement of knowledge in the field of server management inside queueing systems.

. Tian and Zhang conducted a study that examined a queueing system of the GI/M/c type, incorporating the concept of vacations, wherein all servers collectively cease operation when the system is devoid of customers. The authors proposed the notion of synchronous vacations, wherein servers resume operation if there are clients

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in a waiting state. The length of vacations is determined by a random variable that follows a distribution characterised by its phase. The research was centred on the computation of stable probability distributions pertaining to wait durations and queue lengths during arrivals. The work aimed to provide explicit formulas for both measurements.

In their 2002 work, Tian and Zhang examined a GI/Geo/1 queuing model in discrete time, specifically focusing on the presence of server downtime and vacations. The researchers employed a matrix-geometric technique to explicitly compute stationary distributions for both queue length and waiting time.

In their study, Nosek et al. 21 examined queuing-based methodologies in the field of healthcare administration. The authors placed particular emphasis on the evaluation of hospital practises and the enhancement of pharmaceutical services as means to augment consumer satisfaction.

In the study conducted by Katayama 22 the primary focus was on a tandem queue system that incorporated cyclic services. The investigation took into account the presence of servers on vacation as well as a whole service load. The objective of the study was to determine the mean durations of stays, accounting for breaks, as well as the mean waiting times, which are relevant for the examination of performance in packet switching systems.

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Table 1: Summary of Reviews

Study	Sector	Queuing Model	Objective	Methodology	Key Findings
Sasi et al. (2023)	Various (FPS, banks, post offices)	M/M/C	Optimal equilibrium between wait times and server utilization	Simulation modeling	FPS optimization using M/M/C model in Kerala, India
Anand and Arora (2019)	Indian banks	Article	Assess customer service and waiting periods	Case study	Identify and analyze bottlenecks for improving customer service
Bhardwaj et al. (2019)	Communication networks	Fuzzy queuing model	Optimize communication	Article	Voice packetized statistical

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			networks, focus on voice traffic		multiplexing system
Kiataramkul and Neamprem (2019)	Banks (token systems)	Article	Effectiveness of queuing model with multiple servers	Article	Analyze variables relevant to service in different configurations
Onoja et al. (2018)	Banking	Exponentially distributed model	Analyze waiting times, service rates, and arrival rates	Mathematical model	Enhance resource allocation and minimize customer waiting times
Jhala et al. (2017)	Supermarkets	Single vs. multiple queues	Maximize efficiency of queue management	Article	Single queue, multi-server method minimizes client wait times
Thangaraj and Rajendran (2017)	Queueing system with batch arrivals	Article	Investigate scenarios with bulk service	Article	Assess queue size distributions and performance indicators
Agyei et al. (2015)	Banks	Many servers in a single line	Optimize teller staffing for cost-effectiveness	Queuing model	Five-teller system demonstrated superior performance
Harley et al. (2014)	Nigerian banks	Article	Effects of customer service components on financial performance	Article	Relationship between client waiting durations and financial viability
Chandra and Madhu (2013)	Markovian queuing system	Multiple service counters	Analyze distribution of queue sizes	Article	Emphasize state-dependent rates in multi-counter system models
Kamau (2012)	Commercial banks in Kenya	Article	Relationship between waiting queue management and customer satisfaction	Article	Examine strategies, obstacles, and effects on customer satisfaction
Ohaneme et al. (2012)	Petrol stations	M/M/6 queuing system	Evaluate queuing systems at petrol stations	Article	M/M/6 queuing system improves client services

Banik et al. (2011)	Queuing systems (GI/M/1/N and GI/M/1/∞)	Single working vacation policies	Assess effects of server vacations on queuing systems	Markov chain	Evaluate queue length distributions and performance metrics
Rao et al. (2011)	Communication networks	Article	Queuing theory in communication networks	Article	Construct a comprehensive model for an interdependent communication network
Jacob and Szyszkowski (2009)	Call centers	Poisson distribution	Analysis of call center data	Empirical findings	Poisson distribution suitable model for call centers
Adeleke (2009)	University health centers	Single-server queuing model	Estimate waiting times in emergency departments	Article	Significant value in augmenting efficiency of healthcare systems
Cochran and Roche (2009)	Hospital bed shortages	Queuing theory-based models	Mitigate hospital bed shortages	Article	Queuing theory-based models more effective than formula-based techniques
Kasum et al. (2006)	Banks in Nigeria	Article	Comparative analysis of queuing efficiency	Inverted-funnel questionnaire	Consumers of contemporary banks experience reduced waiting durations
Pei-Chun et al. (2006)	Taiwanese banking institutions	Queuing model	Efficacy of different banking institutions	Article	Augmenting the number of ATMs can mitigate consumer wait times
Green et al. (2006)	Healthcare systems	M/M/s queuing model	Interplay between service delays, patient utilization, and servers	Article	Valuable contribution to optimizing healthcare services
Tian and Zhang (2006)	Queuing system with vacation policy	(d, N)-threshold policy	Determine ideal values for variables d and N	Article	Enhance knowledge in server management in queuing systems

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Tian and Zhang (2003)	GI/M/c queuing system	Synchronous vacations	Computation of stable probability distributions	Article	Provide explicit formulas for wait durations and queue lengths
Tian and Zhang (2002)	GI/Geo/1 queuing model	Discrete time	Explicitly compute stationary distributions	Matrix-geometric technique	Examining server downtime and vacations
Nosek et al. (2001)	Healthcare administration	Article	Evaluate hospital practices and enhance pharmaceutical services	Article	Augment consumer satisfaction
Katayama (1995)	Tandem queue system	Cyclic services	Determine mean durations of stays, accounting for breaks	Article	Relevant for examining performance in packet switching systems

CONCLUSION

The findings from these studies underscore the versatility and effectiveness of queueing theory in optimizing service systems. From banking to healthcare and beyond, the application of queuing models provides actionable insights for improving customer satisfaction and operational efficiency. The studies reviewed reveal the importance of tailored queuing strategies, considering factors like server configurations, arrival rates, and waiting times in specific service contexts.

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Volume 1 Issue 1 January 2025

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THE TRANSFORMATIVE ROLE OF LINEAR PROGRAMMING IN CEMENT INDUSTRY OPTIMIZATION

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ABSTRACT

This article explores the application of linear programming in cement industry, focusing on production planning, inventory management, supply chain optimization, energy management, and resource allocation. By analysing case studies and conducting a literature review, the research emphasises the efficacy of linear programming in improving operational efficiency and decreasing expenses. The paper presents a case study that examines the allocation of trucks and shovels in a cement quarry, showcasing concrete advantages. A hypothetical case study concerning the blending of raw materials in an Indian cement facility further demonstrates the adaptability of linear programming. The significance of its advantages is underscored in the conclusion, which also proposes avenues for further investigation concerning optimisation algorithms and environmental impact assessments.

Key words

Linear Programming, Cement Industry, Production Planning, Inventory Management, Supply Chain Optimization, Energy Management, Resource Allocation, Optimization, Case Study.

INTRODUCTION

In the optimisation process, linear programming is a mathematical technique in which the objective function, which is linear, is either maximized or minimized with respect to a set of linear constraints. Linear programming can serve a multitude of purposes within the cement industry:

1. Production Planning: By determining the optimal combination of raw materials to produce a specified quantity of cement while considering constraints such as resource availability, production capacity, and quality standards, linear programming can assist in optimizing the production planning process.

2. Inventory Management: It facilitates the optimization of raw material, intermediate product, and finalised product inventory levels. Thus, storage expenses are minimized and production meets demand.
3. Supply Chain Optimization: The utilization of linear programming can be implemented to optimize various aspects of the supply chain, encompassing cement product transportation, logistics, and distribution. This facilitates the reduction of transportation expenses and guarantees punctual deliveries.
4. Energy Management: The manufacturing process of cement consumes a significant amount of energy. Energy efficiency can be increased through the use of linear programming to determine the optimal combination of energy sources and manufacturing processes.
5. Resource Allocation: With the aid of linear programming, resources such as labour, equipment, and basic materials can be allocated more efficiently in order to meet production goals and reduce expenses.

OBJECTIVE

To assess the impact of linear programming on optimizing key aspects of the cement industry, including production planning, inventory management, supply chain logistics, energy consumption, and resource allocation.

RESEARCH METHODOLOGY

A specific case study is conducted on a cement quarry operation to demonstrate the optimization potential of linear programming in cement industry material allocation.

REVIEW OF LITERATURE

As stated by3 this study develops a method for determining the optimal raw material mixture for an ASCOM cement facility in Egypt by utilising linear programming. This variety conforms to Egyptian chemical composition standards for raw material utilised in cement factories (e.g., 82.5% calcium carbonate, 14.08% silica, 2.5% alumina, and 0.92% iron oxide). In addition, industry-specific parameters (such as lime saturation factor, silica modulus, alumina modulus, and ignition loss) constrain the model. The findings demonstrate that the model effectively replicates the blending process of premium feed containing different proportions of constituents. Additionally, it possesses the ability to acertain the additive limitations of every component. In addition, it illustrates the efficacy of short-term planning for additive procurement and capping limestone quality in order to accommodate variable component combinations. Furthermore, an increase in the quality of the raw blend decreases the limestone feed quality by 50.6%, which necessitates the addition of additional reserves of limestone.

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Following 1 the purpose of the current investigation is to assess the gravel's quality in order to determine whether it could be utilised as aggregate (raw material for roads and concrete). An examination was conducted on the petrographic, physical, mechanical, and chemical characteristics of the sediment samples. Our samples are classified into two groups, carbonate and quartzite, in accordance with ASTM standard 295. Predominant among these samples were those composed of quartzite. The petrographic analysis performed on the gravels revealed the presence of alkali carbonates, opal, tridymite, chalcedony, and crystobalite. The reaction between these minerals and the alkalis in cement causes concrete to expand and fracture. Additional constituents, including sulphides, sulphates, halites, iron oxides, clay minerals, and anhydrites, which may exist in the form of impurities and coatings, are investigated. The results of the current investigation demonstrate that every sample is appropriate for the production of concrete and identify the most economical method for transporting these materials from quarries to cities in accordance with the Egyptian Code.

As stated by 2 the number and variety of linear programming applications to industrial problems have increased at an accelerated rate, rendering it nearly impossible to stay updated on them. This is due to challenging conditions under which many of these applications are executed as well. Frequently, industrial (and governmental) secrecy prevails. Additional restrictions impede the ability to determine and evaluate the pattern of applications. One is the absence of a publication tradition. Failure to determine the overall significance of specific findings is an additional factor, while being disheartened by the publication of comparable applications by others can also be discouraging. Prompt solutions are not readily accessible for these challenges. Presumably, such conventions will benefit over time by fostering informal interactions among individuals who share a common interest.

4 the prevailing mode of raw material transportation employed in cement quarry operations is the truck and shovel. A significant obstacle encountered in cement quarry operations pertains to the effective distribution of shovels and trucks to the mining faces. With quantity and quality constraints in mind, the mixed integer linear programming (MILP) model for truck and shovel allocation to mining faces for the cement quarry is presented so as to minimise the operating costs of trucks and shovels. The GLPK (GNU Linear Programming Kit) standalone solver and the optimisation IDE utility GUSEK (GLPK under SciTE Extended Kit) are utilised to implement this model. An application of the MILP model is made to an established cement quarry operation, using the Kohat cement quarry in Kohat, Pakistan, as a case study. Upon analysing the outcomes of the pertinent case study, it becomes evident that substantial improvements can be attained by implementing the MILP model.

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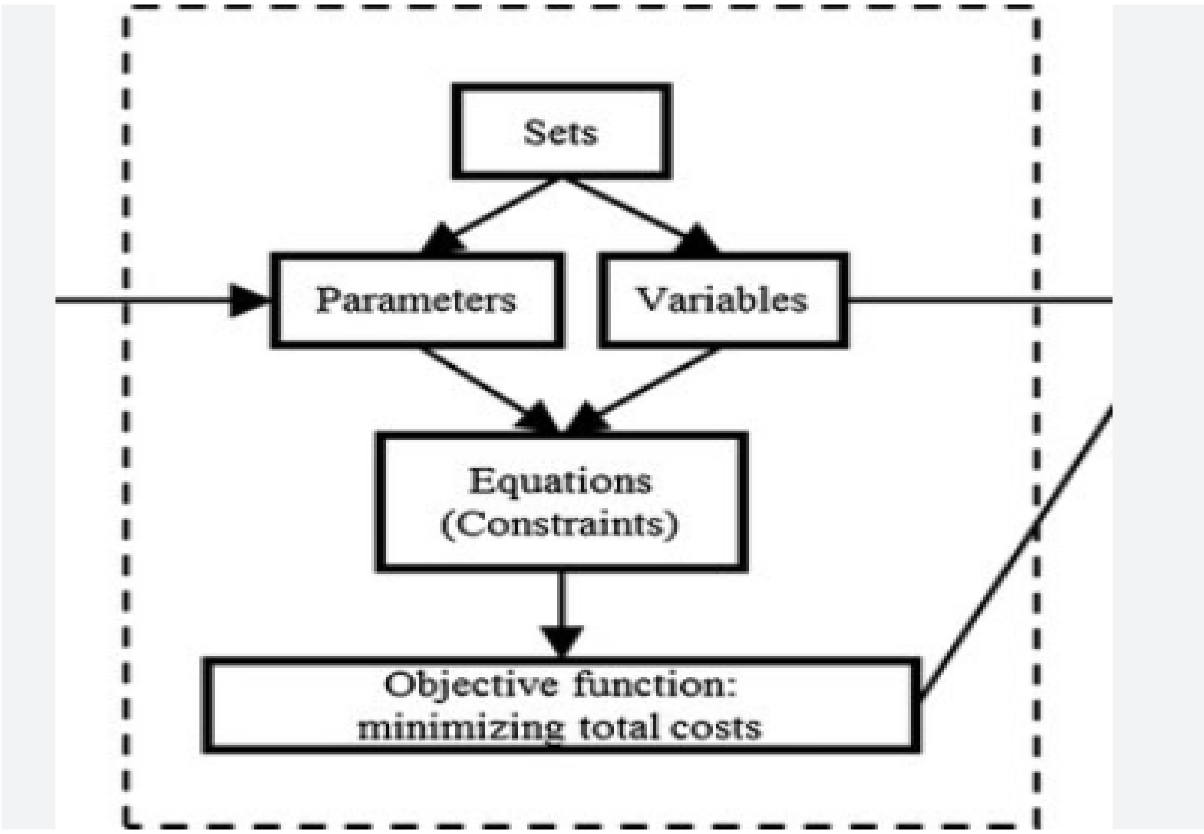
The obtained results not only demonstrate a substantial reduction in expenses but also contribute to improved coordination between the quality department and the quarry.

STRUCTURE OF THE LINEAR PROGRAMMING



Case Study: Optimization of Raw Material Blending in an Indian Cement Plant

Objective: The aim of this study is to enhance the efficiency of raw material compounding in an Indian cement manufacturing facility so as to reduce production expenses without compromising quality standards.

Methodology: The research employs mixed-integer linear programming (MILP) techniques to construct a mixed-integer linear programming (LINEA) model with the aim of optimising the process of combining raw materials. Utilising the GUSEK optimisation IDE and the GLPK standalone solver, the optimisation is executed. The case study is carried out at an Indian cement manufacturing facility, considering information pertaining to the quality standards, chemical composition of raw materials, and production capacity.

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- Chemical composition criteria for basic materials that conform to Indian standards are among the data considerations.
- Attributes unique to the industry, including but not limited to the loss of ignition, silica modulus, lime saturation factor, and alumina modulus. • Limitations on production capacity and quality requirements.

Findings: The analysis provides evidence that the MILP model optimises the merging of raw materials in an efficient manner, leading to decreased production expenses and enhanced interdepartmental coordination at the facility. Through the implementation of quantity and quality limitations, the model effectively optimises resource utilisation and guarantees that the ultimate cement product conforms to the prescribed specifications.

The utilisation of linear programming to optimise the blending of basic materials in the Indian cement industry is exemplified in the case study. The results indicate that comparable methodologies could be implemented in additional facets of cement production, thereby resulting in financial savings and improved operational effectiveness.

Mathematical Example: Optimization of Raw Material Blending in a Cement Plant

Let's consider a simplified mathematical example to illustrate the optimization of raw material blending in a cement manufacturing plant. The objective is to minimize the production cost while meeting specific chemical composition criteria and production capacity constraints.

Variables:

- Let X_1 represent the quantity of raw material A (e.g., limestone) to be used in the blend.
- Let X_2 represent the quantity of raw material B (e.g., clay) to be used in the blend.

Objective Function: Minimize the production cost, which is a linear combination of the costs of raw materials A and B:

Minimize $Z = (c_1\, x_1 + c_2\, x_2)$

Where:

- C_1 is the cost per unit of raw material A.
- C_2 is the cost per unit of raw material B.

Constraints:

1. **Chemical Composition Criteria:**
    - $0.80X_1+0.10X_2 \geq 0.75$ (Requirement for Calcium Carbonate)
    - $0.05\ X_1+0.70\ X_2 \geq 0.65$ (Requirement for Silica)
    - $0.05\ X_1+0.20\ X_2 \geq 0.15$ (Requirement for Alumina)
2. **Production Capacity Constraint:**
    - $X_1 + X_2 \leq 1000$ (Total quantity of raw materials should not exceed the plant's production capacity).
3. **Non-negativity Constraints:**
    - $X_1 \geq 0$
    - $X_2 \geq 0$

Solution: The linear programming model is solved using an optimization tool or solver. The optimal values for X_1 and X_2 will provide the most cost-effective blend of raw materials while satisfying the chemical composition criteria and production capacity constraints.

Note: The coefficients and constants in the objective function and constraints would be based on the specific costs of raw materials, chemical composition requirements, and production capacity of the cement plant in the real-world scenario. The example above is a simplified illustration for conceptual understanding.

For further research, it is recommended to explore the dynamic aspects of raw material availability, considering variations in market conditions and supplier constraints. Additionally, the environmental impact of optimized blending processes can be a subject of investigation, aligning with sustainability goals in the Indian cement sector.

CONCLUSION AND SUGGESTIONS

In conclusion, the application of linear programming in the cement industry proves to be highly beneficial for optimizing production processes, reducing costs, and enhancing overall operational efficiency. The case study demonstrates tangible benefits, indicating that similar approaches can be applied to other aspects of cement manufacturing.

Suggestions for further research include exploring the integration of advanced optimization algorithms, considering dynamic constraints, and conducting in-depth analyses on the environmental impact of optimized processes. Implementing such suggestions can contribute to the continuous improvement and sustainability of the cement industry.

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